

# Does God practice a random walk? The ‘financial physics’ of a nineteenth-century forerunner, Jules Regnault\*

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L'homme s'agit, Dieu le mène.

(Jules Regnault 1863: 52)

## Introduction: on secret stories

Generally speaking, histories of the random walk hypothesis in finance – forerunner of the efficient capital markets model – repeat the following story: in 1900, an unrecognized genius, Louis Bachelier, conducted a study of French government bonds and found that their price changes were consistent with a random walk model.<sup>1</sup> ‘Besides anticipating the empirical work that was to come more than half a century later, Bachelier also developed many of the mathematical properties of Brownian motion (the continuous-time analogue of the random walk) which had been thought to have been first derived later in the physical sciences. In particular, Bachelier had anticipated many of the mathematical results developed in Albert Einstein’s 1905 paper’ (LeRoy 1989: 1587); ‘the study of . . . stochastic models of price behavior is not at all new, since many of the fundamental techniques of random processes happen to have been first considered in the context of economics, in 1900, by Louis Bachelier’ (Mandelbrot 1966: 243); ‘though his contributions were ignored for sixty years, the first statement and test of the random walk model was that of Bachelier’ (Fama 1970: 389).

However, we suspect that such an identification of the origin of the random walk hypothesis in finance, and consequently of the modern

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approaches of capital markets, is historically imperfect and mistaken. In this paper, we indeed claim that in his *Calcul des Chances et Philosophie de la Bourse* (1863), a French forgotten economist, Jules Regnault, laid the basis of modern stochastic models of price behaviour. Straight away, we have to confess that his saga remains perfectly secret: no information is available on the various aspects of his life and work.<sup>2</sup> Our research only led us to believe that his book circulated among a small circle of French economists, engineers and actuaries, like Emile Dormoy, Maurice Gherardt and Hermann Laurent; however, we have found no reference to the book after 1923. It thus seems that he inaugurated the path to non-recognition that was subsequently to be followed by Bachelier. This lack of recognition was certainly unfair, since the book deserved close attention and contains many original ideas: for instance, Jovanovic (2000) shows in a companion paper that the great part of Bachelier's reasoning came directly from Regnault – most importantly the random walk hypothesis.

In this paper, however, we shall not develop such a genealogy and filiation explicitly. Nor shall we claim that the identification of a founding father is important *per se*. We rather think that a study of Regnault's work in its own context can be fruitful: we shall indeed suggest here that he was a highly typical figure of the mid-nineteenth century. For instance, his approach to financial markets was deeply rooted in harsh debates that occurred in France during the period: at a time when speculation was morally denounced, Regnault undoubtedly followed Quetelet's programme of social physics,<sup>3</sup> designed to form the foundation for an exact science of human societies, and approached 'scientifically' the issue of speculation. His work tried to show that short-term speculation – based on the search for immediate gains – leads to ruin but that, by contrast, another kind of speculation, based on the long-term gains, was socially useful. Interestingly, this piece of work relies on a typically nineteenth-century philosophical and methodological framework: it belongs to the deterministic paradigm, based on the belief that exact laws rule the phenomena and could be unveiled by mean values – Regnault was more interested in mean values than in variation for its own sake, seen as kinds of vices. As other economists and social scientists of that time, he also perceived analogies with the natural sciences and imported some methods, concepts and laws from these disciplines. But his approach to economics was also certainly pioneering: as early as 1863, his work took the shape of theoretical and mathematical models – that were however discussed in literary terms<sup>4</sup> – and, at a time when the application of statistics to economic affairs was controversial in France, some of these models were combined with statistical procedures. In other words, we suggest that a historical analysis of Regnault in his own context leads us to think that this author sheds a new light on

the origins and early uses of economic models, of econometrics, and of course of financial theory.

This paper is organized as follows. Part one explains Regnault's aim and framework. At a time when short-term speculation was denounced as immoral, he approached this issue 'scientifically' and constructed models that separated long-term speculation from the desire for immediate gains. Moreover, his conception of Nature as a simple and divine machine motivated the use of laws devised by natural scientists. Part two analyses his first model, which aims at analysing short-term speculation. It first took the shape of a random walk model but the introduction of transaction costs enabled him to show that short-term speculation leads to bankruptcy. Finally, part three presents Regnault's model of long-term speculation, which aims at evaluating the mean value of the French 3 per cent bond.<sup>5</sup>

## 1. Regnault's framework: touched by the hand of God

In this first part, we suggest that Regnault's work cannot be separated from the context of his time. First, the book was stimulated by debates on speculation in France – which we shall briefly expose – and it can be seen as a 'scientific' answer to problems that the jurists and the economists of the former generation could not solve. Second, we present his framework which was highly representative of the scientific and deterministic views of the mid-nineteenth century; we shall also explain why he thought that natural phenomena and human phenomena were governed by similar laws.

### 1.1 *The historical context: sounds of fury*

*Speculation in France: a bitter sweet symphony* Following proposals by influential economists, futures markets were legally recognized in France on 28 March 1885.<sup>6</sup> However, for a long time, attitudes towards them were divided: they were perceived as suspect but at the same time exerted a strong attraction over individuals as well as over successive governments. From the beginning of the eighteenth century to the second half of the nineteenth century, there was a conflict in France between the economic need for futures markets and moral issues, and a large variety of means that aimed at warding off their dangers and abuses were experimented in the country. This conflict can be illustrated by the fact that during the nineteenth century, a gap constantly increased between a restrictive legislation – decided by the French government – and jurisprudence, which took into account the increasing financial needs of the time.

An origin of this history can be identified: the collapse of Law's system

in 1720. This system was initially supported by the French government – in the hope it would lead to a reduction of its debt. However, it rapidly initiated a speculative phenomenon that ended up with a financial crash. The legislation immediately and severely denounced the presumed culprit: the speculation [*agiotage*.<sup>7</sup> This repressive legislation was motivated by reasons of both a moral and a political nature: on one hand, the government denounced enrichment based on a kind of game; on the other hand, a condemnation of such a practice aimed at a purification of the futures markets that were a means for the financing of the French public debt: at that time, these markets were mainly composed of public bonds, and any lack of public confidence had to be avoided.

However, it soon appeared that this repressive legislation was inadequate, in the sense that several kinds of speculation were condemned although they were economically useful and even necessary.<sup>8</sup> The courts scrupulously respected the law as soon as financial scandals occurred, but were also interpreted in a more flexible way and never condemned the operations that favoured public bonds. In a more prim fashion, the courts were thus led to search for criteria that were necessary to characterize various kinds of operations on the stock markets and thus to identify those which should be condemned. A distinction was first established between short purchase and short sale:<sup>9</sup> the former was seen as helping to increase the price, and thus as favouring public bonds, whereas the latter was condemned. Then, in a second step, it appeared that short sales were economically necessary and were also of different kinds: it was thus eventually decided to make a distinction between licit and illicit operations<sup>10</sup> – between the speculation and the game – on the basis of criteria such as the wealth of the agents or the opinion of the *agents de change*.<sup>11</sup>

*Economists on the forefront: scientists versus flat-earthers* Economists were involved in these debates<sup>12</sup> and exerted a strong influence on jurisprudence, as they contributed to the construction of the above-mentioned typology of operations.<sup>13</sup>

However, these issues initiated debates among two traditions of economists. On the one hand, that of Alphonse Courtois 'fils', an influent economist who was highly representative of the economists of the time.<sup>14</sup> From a theoretical point of view, he was a liberal economist and a member of the French Classical tradition – his ideas were close to those of Say. He claimed the usefulness of the stock market as a whole. From a methodological point of view, his approach was purely literary and descriptive and was characterized by a rejection of mathematics, probability and statistics. However, on the other hand, the mid-nineteenth century was characterized by the emergence of other approaches to these issues.<sup>15</sup> Authors such as

Henri Lefèvre<sup>16</sup> and Regnault claimed the need for ‘something new’: the construction of a ‘scientific’ approach to the stock market.

The existing books on Stock markets and banks are of no help towards the understanding of the very mechanism of their operations, although such was their aim. . . . The *Traité des Opérations de Bourse*, by Courtois, is of no help to operate; . . . no precise idea that could help towards teaching or a scientific approach can be found here.

(Lefèvre 1885: III)

On theoretical grounds, both authors stressed the need for an understanding of the very nature of the way the stock market was functioning: their aim was not to present a collection of examples, but to identify laws that ruled this market. On methodological grounds, they rejected the literary style and approached the problem ‘scientifically’: Lefèvre followed a graphical path while Regnault, seven years earlier, followed a statistical and probabilistic path.

The aim of our hero was the identification of ‘new laws of the variations on the stock market’ (1863: 7). It has to be noted that he saw his investigations as undoubtedly scientific – his agenda was the construction of a ‘Science of the stock market’ (1863: 47). The book is closely linked to nineteenth-century debates: the goal was to ‘demonstrate the dangers of the game [*jeu*] and to discover what Speculation should propose’ (1863: 1). But the approach was a pioneering one: it was not based on moral presuppositions *per se* but on a rational demonstration of the consequences of immoral behaviour of individuals – driven by their sole ‘personal interest’ (1863: 1) – on society as a whole as well as on individuals, proving that such a behaviour led to their inexorable ruin. He indeed believed that unlike morals as such, a ‘scientific proof’ was definitively convincing:

Until now, the various forms of morals were used to criticize the abuses of speculation, and to try to correct them; but morals, in order to persuade, has to convince . . . ; to be convincing, the truths have to be obvious, irrefutable.

(1863: 1–2)

His aim was thus, in a scientific perspective, to separate two kinds of speculation: short-term speculation and long-term speculation.<sup>17</sup>

We have seen that Regnault’s agenda is rooted in debates that occurred in France since the eighteenth century. We now have to analyse more completely his framework, which is, to some extent, highly representative of some scientific views of the mid-nineteenth century.

## 1.2 Regnault’s methodological framework and worldview

*On God’s clockworks* Regnault believed that the terms ‘scientific’ and ‘deterministic’ were synonymous and inseparably connected – he undoubtedly

had good reasons to refer to Laplace in the early pages of his book (1863: 7). His fundamental postulate was that chance does not exist: 'in nature, nothing is arbitrary' (1863: 2). Nature was characterized by 'general and unchanging [*immuables*] laws' (1863: 7), society was 'a huge machine made up of springs that are connected' (1863: 202): both were precision clocks that are regular, orderly and highly predictable. Four features of this deterministic framework deserve attention.<sup>18</sup>

First, these deterministic views are, of course, associated with the issue of causality. Regnault constantly argued that 'nothing can happen without a previous cause' (1863: 2) and that 'anytime, everywhere, the same causes produce the same effects' (1863: 135). The *Calcul des Chances et Philosophie de la Bourse* was then based on a basic distinction between two kinds of cause: 'accidental' causes and 'constant' causes – a distinction inherited from Laplace.<sup>19</sup> The former were produced 'without any apparent law and fortuitously' (1863: 11) and were the concern of the short term. As far as the stock market is concerned, such causes were new and exogenous information. By contrast, the 'constant causes' were acting 'continuously and regularly, always in the same direction and with the same intensity' (1863: 11). They were associated with long-term trends and linked to exact laws, although masked by the existence of accidental causes at work.

Second, the deterministic laws were only accessible to us in a state of perfect knowledge: it would then be possible to discover what the future had in store for us. However, individuals have bounded knowledge and have to content themselves with an approximate knowledge of these laws. This imperfection of knowledge has two consequences. On the one hand, individuals can only try to tame the future through the elaboration of probabilities that are, of course, of a subjective nature: 'Chance does not exist, only our ignorance exists' (1863: 2). 'Our whole calculations are only based on our *personal observation*' (1863: 18). On the other hand, he emphasized that the underlying uncertainty favoured errors and deviations: 'Ignorance, that . . . maintains our illusions and errors, is the first cause for our excesses [*débordements*], our passions, our misfortunes' (1863: 2).

Third, science was seen as a means to approach a harmonious state of perfect knowledge – the aim of science was the constant reduction of 'the limits of doubt' (1863: 2), through the discovery of the mechanisms that rule the universe. In that perspective, his goal and his methodological framework – his greater interest in mean values than in variations – were well matched. On one hand, deviations from the mean were associated with accidental causes and considered to be errors and moral vices; error was thus banished from the universe. On the other hand, the mean value represented a harmonious equilibrium in the social world, the ideal in morals:<sup>20</sup> it was the sign for an order that he supposed to prevail beneath

the whirl of observed phenomena. The mean was seen as the manifestation of the deterministic laws that rule natural and human affairs, and statistics was seen as a means to approximate determinism. Once these exact laws were discovered by scientists, the individuals could then reach ‘a stable and quiet state’ (1863: 163). Deviations should thus be reduced through a progress of civilization or of advancement in scientific knowledge; men could then behave in conformity with God’s will.

Fourth, and finally, Regnault saw the underlying order and structure of the universe – from the formation of prices on the stock market to the trajectory of planets (1863: 98) – as created by God. He constantly referred to the existence of ‘superior and providential’ laws (1863: 185) that each individual should respect. Indeed, man was or should be subordinated to God’s laws:

What a subject for amazement and admiration are offered by the views of Providence, what reflections are suggested to us by the marvelous order that characterizes the least detail of the most hidden events! What! The variations of the Stock market are ruled [*soumises*] by constant mathematical laws! Events that are generated by the caprices of men, the most unpredictable shocks of politics, the most cleverly studied financial combinations, the result of a multitude of events that are not related, all these effects are tied up in an admirable set [*ensemble*], and *chance* is now a meaningless word. And now, you, the princes of the Earth, have to learn and to be humble, you who in your pride, were dreaming of a mastery of the People’s destiny; you, the kings of finance, who have at your disposal the wealth and the credits of nations, you are nothing else than frail and docile instruments in the hands of the *One who masters [embrasse] the whole causes and the whole effects in the same order*, and who, according to the expression of the Bible, has measured, calculated, evaluated and distributed everything according to a perfect order. The men are stirring, God rules them.

(1863: 51–2)

There is no doubt that Regnault was touched by the hand of God: his identification of a divine law ruling financial affairs should pave the way for the speculator’s happiness and for the stability of society – ‘the discovery of the future . . . leads to wisdom’ (1863: 140).

*A unified conception of the world: analogies and transfers at work* Another feature of Regnault’s deterministic framework is to be found in the idea of reductionism: the various bodies that constitute the universe were seen as ruled by the same kinds of laws. A unified conception of nature means here that some laws were constantly at work, through time and through space, and at very different levels, and that various sciences and disciplines have to share principles, methods and also laws in common. It is well known that several social scientists of the nineteenth century and of the turn-of-the-century era developed similar ideas, and were consequently led to approach economics at the light of the natural sciences.<sup>21</sup> Regnault is a

perfect illustration of this reductionism: he indeed claimed that ‘the moral world is ruled by the same laws as the physical world’ (1863: 5).

Consequently, he looked for financial translations of existing natural laws and identified some analogies with the natural sciences. We shall not discuss here the basic functions of analogies – i.e. the fact that they are a prolegomenon to thought, are largely motivated by prestige and aimed at persuasion.<sup>22</sup> We have here a predilection for another kind of historical explanation: it seems to us that the analogies perceived by Regnault are close to what was labelled by Israel the ‘mechanical analogies’ which took place in a unified conception of the world:

Science has to construct a unitary and objective picture of the universe. . . . The various parts of science, as well as the theories which apply to various kinds of phenomena, have to be connected and to be consistent.

(Israel 1996: 19)<sup>23</sup>

The laws identified by natural scientists would thus have counterparts in the social world and could be transferred into that field: God created both the natural and the social worlds according to similar principles. This feature is exemplified by the testing procedure he used: Jovanovic (2000) argued that this procedure was original, in the sense that it included two steps. First, an ‘economic test’ was based on a comparison of theoretical laws with observed data – this test is sometimes close to an econometric one. Second, a ‘non-economic test’ was based on a comparison of the theoretical laws with existing natural laws; this was a means for Regnault to ensure that his own work in the social field was connected to fundamental wheels of nature.

## **2. The model of short-term speculation**

The aim of the first model<sup>24</sup> – that he labelled the model of game (*‘le Jeu’*) or the model of short-term (*‘courte période’*) speculation – is to demonstrate that this kind of speculation inexorably leads to ruin. First, Regnault built a symmetrical random walk model, based on two hypotheses (relative to the arrival of new information on the market and to the subjective interpretation of this information), and he then deduced a ‘law of deviations’ that he tested. Second, he introduced transaction costs in the model and put forward the ruin of the short-term speculator.

### *2.1 The random walk model*

*On random walk: Regnault ahead* The aim of Regnault was first to construct a model of short-term speculation. In that perspective, he analysed the stock market at the light of ‘a game of heads or tails’ (1863: 34). This choice

deserves two comments. First, his aim was to approach the variations of the prices in a probabilistic perspective, and the main interesting feature of the model is precisely a modelling of price behaviour that is similar to the binomial probabilistic model of a game of heads or tails:

On the stock market, the whole mechanism of the game comes down [*se résume en*] to two opposite possibilities [*chances*]: increasing and decreasing. Each one can always appear with an *equal facility* [*facilité*].

(1863: 34)

Second, this choice was also motivated by moral considerations: Regnault aimed here at constructing a model of moral short-term speculation, i.e. at showing that speculation is moral because each agent has here perfectly equal chances to make profits – ‘at any moment, no advantage is existing for one possibility or for the other’ (1863: 34). In such a case, the expected profit is zero for each operation<sup>25</sup> and consequently he aimed at showing that the stock market should not to be condemned *per se*. In a very pioneering way, price behaviour took the shape of a random walk model – although he never used the word.

This was undoubtedly an important step in the construction of financial theory as well as in its theoretical basis.<sup>26</sup> Bachelier will follow this path in his 1900 dissertation<sup>27</sup> – in which he first offered a formalization of the Brownian motion and thus initiated the mathematical theory of stochastic processes in continuous time (Mandelbrot 1966). Models of that kind will also be used in the construction and the test of the theory of informational efficiency that was synthesized by Fama (1970). Given this relation between the theory of efficient markets and the random walk model, it is not surprising to discover in Regnault’s book the main lines of the contemporary theory of informational efficiency of financial markets: information as well as its interpretation were seen as two kinds of causes that generate the short-term variations of stock prices.

*The movement of prices and the arrival of new information* An important issue at work in this book is indeed that of information. The short-term speculator is only interested in immediate profits – more precisely, profits that are made during a period which is inferior to the ‘liquidation’.<sup>28</sup> He thus tries to gain from any variation of prices – i.e. from accidental causes. Regnault’s originality lies in this association of these causes with the new information that arrives on the market. In addition, he claimed that at every moment of time, the price contains the whole information:

When the speculator [*joueur*] buys or sells, expecting an increase or a decrease of the price [*cours*], he thinks that the price is below or above the true *price* [*valeur*]: because for this determination, it is necessary for him to perceive in the current situation a cause of increase or decrease which is not yet taken into account. He could not

pretend that he anticipated an increasing or a decreasing on long term consequences. We know that these consequences, if they exist, are contained in the current price. However, from the very meaning of the word *value* [*valeur*], we see that the value is and can only be determined by the very *price* [*cours*].

(1863: 29–30)

The price of a security thus contains the whole information and its variations result exclusively from the arrival of new information on the market. From this idea, Regnault identified two assumptions relative to the distribution and the independence of the variations that he translated into the language of probability.

Indeed, he explained that ‘on the stock market, the entirety of possible events can only determine two opposite effects: *increasing* and *decreasing*’ (1863: 15). From his conviction that the price contains all the information and that its variations result from new information, he deduced that the probability for the price to increase is equal to the probability for the price to decrease, i.e. a half. If such was not the case, agents could arbitrage – a word explicitly used by Regnault (1863: 40) – and choose systematically the strategy that has the greatest probability:

In all the games of chance that contain two opposite chances, relative equality precisely results from the possibility for the player to choose one chance or the other: moreover, these two conditions cannot be separated, because if a possibility would generate a greater advantage than the other one, it would be constantly chosen.

(1863: 41)

Regnault also claimed that, like in a game of heads or tails, the movements of stock prices are independent: neither the variations, nor previous prices were useful to predict future variations:

When I toss up, it is certain that each toss is completely independent from the previous ones. . . . In a similar way, the speculator on the stock market is always trying to make forecasts [*conjecturer ce qui doit arriver*] on the basis of what has already happened, in such a way that after three or four days of decreasing, he will be led to anticipate an increase for the next day, or, at the opposite, will be led to think that the decrease tendency will last; although a complete independence is existing between these effects.

(1863: 38)

Otherwise stated, the only thing that agents know for certain is the fact that, at every moment of time, the price can increase or decrease with the same probability of a half, independently from the previous prices. Consequently, the expected profits for each agent are zero. From these two assumptions (an equal probability and the independence of the variations), Regnault was thus moving towards an analysis of the price behaviour in terms of a random walk. However, another particular feature of the short-term behaviour is the subjective nature of the probabilities.

*The movement of prices and the subjective evaluation of new information* The subjective nature of the probabilities affects the evaluation of securities as well as their price, but Regnault suggested that the evaluations made by individuals were following a normal law.

Indeed, and despite the fact that public information is known by every agent, the evaluation of the consequences of any accidental cause on the price remains particular to every agent: ‘There will perhaps not be two speculators among one thousand that have the same opinion on all the causes and their effects’ (1863: 20).

Of course, this variety of opinions makes possible the exchanges – ‘if everybody . . . evaluates similarly the same causes, no possible counterpart would exist’ (1863: 22). However, we would like to focus on a particular property of these subjective evaluations: they were seen as including some errors but are ruled by a particular law. Regnault justified this idea on the basis of an example, that of the evaluation of the height of a building by a group of individuals,<sup>29</sup> and was led to think that these evaluations were ruled by a law close to the Gaussian one, considered as the law of chance. He then applied this example to the stock market, and considered two groups of agents that were equally distributed around the mean value: the *bulls* [*haussiers*] and the *bears* [*baissiers*], to use our modern vocabulary. This distribution could explain the short-term movements of prices. The structure of this distribution leads, once more, to an equal probability of a half for the prices to increase or to decrease, at each moment of time.

These assumptions thus enabled him to construct a symmetrical random walk model. We must now show the way he obtained from this model a law of variations of the price through time.

*The law of deviations* Regnault thus showed that there was an equal probability of a half for the price to increase or decrease at each moment of time. He then investigated the way the prices were varying through time, and he discovered a relation – also at work in Bachelier (1900) – between the mean deviation of prices and time.<sup>30</sup>

When an agent is involved in a new operation, he is in a relatively uncertain situation: an agent can choose a certain deviation – i.e. the profit he wishes or the loss that he is ready to support – but the time of the operation remains uncertain or, by contrast, he can choose a determined period but an undetermined deviation (1863: 48). In the latter case, a mean deviation for a given period can be calculated, and Regnault looked for a relation between time and that deviation. He remarked that the mean deviations for a given period of time were approximately equal and that the shorter the period of time considered, the smaller the deviations. Moreover,

if the period of time is only half as much, the deviation is divided by a number inferior to two. This observation led him to claim that:

A mathematical law is thus existing and rules the variations as well as the mean deviation of the prices on the Stock market; this law, which apparently has never been discovered, is here presented for the first time: THE DEVIATION OF THE PRICES INCREASES WITH THE SQUARE ROOT OF TIME [*L'écart des cours est en raison directe de la racine carrée des temps*].

(1863: 50)

Otherwise stated, it is not possible for an agent to anticipate in an exact way the future price of a security, but he can anticipate its mean deviation for a given period of time.

Regnault then carefully tried to examine the robustness of this law and he used two kinds of testing procedures: a non economic one, based on a parallel with existing scientific laws, and an economic one, close to an econometric one. In the first part of the paper, we have seen that he believed that the various parts of the world were ruled by similar and universal laws. He thus suspected that the law of deviations was not only at work in the field of finance:

The security [*La valeur*], in its variations, is constantly in search for its true price [*véritable prix*], or an absolute price, that we can represent as the *center* of a circle, the *radius* of which would represent the deviation that can equally appear in the one or the other direction, and on each point of the surface, in a period of time consequently equal to the surface, and the whole points of its *circumference* would represent the limits of the deviations. In its variations, the security [*valeur*] is only moving away or getting closer to the center, and from the basic notions of geometry, we know that the radius or the deviations are proportional to the square roots of the area or of time.

(1863: 51)<sup>31</sup>

Then, he empirically tested the law, using monthly data relative to the 1825–62 period. He determined the mean deviation for various periods (a month, a quarter, a year): for instance, he found that the monthly mean deviation was 2.73 and that the yearly mean deviation was 9.50. He tried to ensure that these deviations were obeying to the theoretical law: he thus verified that 9.50 (the observed mean deviation) was close to  $2.73 = 9.45$  (the theoretical mean deviation). He concluded that ‘the theory and experience are matched’ (1863: 175).<sup>32</sup>

This first model was for him a moral one, in the sense that it is based on equal chances (the price can increase or decrease with the same probability of a half). However, Regnault constructed a second model of short-term speculation that incorporated the transaction costs and that enabled him to demonstrate the inexorable ruin of the short-term speculation.

## 2.2 The introduction of transaction costs

*The nature of the costs* Given this law of price variation, Regnault got interested in the costs that affect the speculator when he trades: the risk of dealing with an informed trader, the risk that the order will not be executed, and the risk that agents manipulate prices or information. However, they were considered as negligible – they can be avoided or reduced, or are kinds of exceptions. For instance, the risk that an agent deals with an informed trader can be reduced by an appropriate choice of bonds – those whose market is large:

Other things the same, the unfavorable possibilities [*chances*] are largely reduced on a large market, where the business is very important, and they proportionally increase on a small and especially a sensitive [*impressionnable*] market.

(1863: 43–4)

Similarly, the manipulation of information was seen as marginal (1863: 97–8), and the risk that the order will not be executed (1863: 43–4) can be reduced by a change in the limits of the order price.

By contrast, a fourth element plays a key role in Regnault's model: the transaction costs [*frais de courtage*] – to use once more the modern vocabulary – that concern every operation and that modify the equal probability to win and to lose a certain amount of money. Indeed, without any transaction cost, the probabilities for the price to increase or to decrease on the average of an amount of  $x$  francs, during a period of time  $t$ , are equal ( $\frac{1}{2}$ ). By contrast, when a transaction cost  $c$  exists, the probability for the price to increase in average of  $x-c$  or to decrease in average of  $x+c$  is  $\frac{1}{2}$ : the mean deviation is thus changed, and the expectancy to win becomes negative ( $-c$ ). In order to get back the same mean deviation  $x$ , it is necessary to have a probability of winning that becomes inferior to the probability of losing. Consequently, ‘The equal chances to win or to lose different amounts of money [leave room to] unequal chances to lose or to win an identical amount of money’ (1863: 212).

But in such a case, the probability to lose a given amount becomes greater than the probability to win that amount and the expected profits become negative. Regnault thus had in his hands the elements that could pave the way for a ‘law of ruin’.

*A law for the ruin of the short-term speculator* Each time an agent transacts, he has to pay a transaction cost. In such a case, his mean loss is equal to the amount of transaction costs multiplied by the number of operations – the frequency of operations being considered as constant. Regnault can then put forward three results. First, ‘the chances of loss are increasing with the power of the amount of wealth [*les chances de perte s'élèvent à la puissance*

*donnée par le rapport direct des fortunes]*’ (1863: 91) – i.e. it is better to have initially a quite low amount of money. Second, ‘the chances of loss are increasing with the power of the inverse ratio of amounts [*les chances de perte s’élèvent à la puissance donnée par le rapport inverse des quotités*]’ (1863: 91) – i.e. it is better to have a predilection of important operations. These two elements are not specific to the financial market: they are relative to every kind of random game. Third, he identified a specific element that exerts an influence on the chance to lose on the stock market: time. Indeed

The chances of loss are increasing with the power of the inverse ratio of time [*les chances de perte s’élèvent à la puissance donnée par le rapport inverse des temps*]’ (1863: 91)

Regnault can then demonstrate that short-term speculation inexorably leads to ruin. On the long term, the sum of accidental causes do compensate *in fine*; the short-term speculator, who constantly wants to benefit from the variations that result from these accidental causes, has to transact rapidly and frequently. However, rapid trades generate only small profits – according to the law of deviations – and frequent trades lead him to lose his capital – because of transaction costs. Consequently, this kind of speculation – based on the search for immediate profits – inexorably fails: the profits cannot compensate for the transaction costs.

Regnault thus ‘scientifically’ demonstrated that the ruin of the short-term speculator could not be avoided and that this kind of practice was a deviation – in the sense of a vice:

*The frequency of operations is an abuse*, and since the unique motivation of each exchange is and has to be *usefulness* [*utilité*], each time this usefulness disappears, there is an error or a bad use; from this, we can thus clearly separate the game [*agiotage*] from the speculation.

(1863: 105)

He thus demonstrated the dangers of the game on the stock market, and from his law he believed that he was able to predict the exact moment when an agent will be ruined:

It is even possible to predict the moment when the short-term speculator [*joueur*] will be ruined. . . . The transaction costs [*droits de courtage*] generally represent % or for the game business and for the serious business. . . . As a result, if the amount of money that he can lose represents for instance the twentieth of the capital that enables him to transact each time, the transaction costs represent for him twenty times more, that is 2.5%. As a consequence, since the gain and the loss equilibrate [*se balancent*], no more than forty operations will lead to the loss of his capital.

(1863: 95)<sup>33</sup>

Given this law of ruin, another kind of behaviour had thus to be thought: long-term speculation.

### 3. The model of long-term speculation: a non-random walk down econometrics

The aim of this third part is to analyse Regnault's second model, that of 'true speculation' ('*la spéculation*' or '*la véritable spéculation*') or of 'capital', which is constructed in the second part of the *Calcul des Chances et Philosophie de la Bourse*. It mainly aims to show that in the long term ('*longue période*'), deterministic laws rule the stock market. We shall first examine the way long-term speculation was defined, the way it was seen as representing a social usefulness, and explain its relations with constant causes. Then, it will be possible to shed some light on Regnault's methodological choices: he thought that the accidental causes compensate on the long term and consequently that the underlying order that rules society could be discovered. In Quetelet's style, his approach was closely associated with the determination of mean values and this was an opportunity to claim the usefulness of a statistical approach to the social world. Finally, we shall present the model, constructed in four stages: it aimed at identifying a new kind of 'attraction' and undoubtedly deserves a particular space in the history of econometrics.

#### 3.1 Some speculation about long-term causal laws

The construction of this second model originates in the belief that true deterministic laws were permanently acting although masked by the accidental causes previously studied in part two of this paper. As seen in part one, these laws were associated with constant causes at work. One can straightaway note that two kinds of constant causes were identified: 'general' ('*générales*') vs 'special' ('*spéciales*') constant causes. The first ones were relative to events that concern the whole general conditions (for instance the whole security market):

They are relative [*concernent l'ensemble*] to a political, commercial and financial situation, they lead to a rise or a decline of the general interest rate, or exert an indirect influence on prices [*prix*] via imposition ... or financial measures whose main effect is an acceleration or a reduction of the circulation of securities [*valeurs*].  
(1863: 111)

By contrast, the 'special' constant causes were relative to a particular security (1863: 111).

Regnault first looked for a special constant cause that 'contributes to the formation of the true price of the bond [*valeur*]' (1863: 111) and that could form the skeleton of a theoretical and simple model. This identification resulted from the main feature of a bond – the payment for an interest, defined as 'the representation of the risk for capital' (1863: 124). He thus

believed that 'the *interest* rate is the most important element in the formation [*constitution*] of a security' (1863: 120). This focus on interest deserves two comments. First, risk was here considered as a source of instability and Regnault thus claimed his predilection for bonds offering low interest: agents have 'to avoid too important risks that can compromise [their] own capital' (1863: 123), since 'numerous dubious [*véreuses*] enterprises offer unjustified dividends [...] as misleading lures [*appât trompeur*]' (1863: 124). Second, a similar idea can also be found in his distaste for variable interest securities: this variability was seen as generating some uncertainty about future dividends and was consequently offering some basis for the game (1863: 122–3). These two comments contribute to the understanding of Regnault's study of a particular security, the French 3 per cent bond: the identification of the constant cause was certainly eased by the constant nature of the interest; moreover, we guess that this choice was an opportunity for him to claim the morality of such a bond.

The knowledge of such a constant cause was indeed delineating the outlines of a moral behaviour that leads to a social usefulness. At that stage, Regnault clearly opposed the short-term game to long-term speculation. The former was largely independent of interest and capital, and consequently resulted in a 'ghost market' which is non-significant as 'a game practiced in a gambling-den [*tripol*] can be' (1863: 108). By contrast, long-term speculation was undoubtedly a fruitful path: it was generating trust.<sup>34</sup> Moreover, this path was a moral one, in the sense that it was associated with labour from two points of view. First, an agent has to spend a considerable amount of time and labour to choose appropriate securities, i.e. to identify those which have serious economic counterparts (1863: 103). Second, these counterparts were precisely seen as sets of labour: 'the profits ... result from an accumulation of Labor, of Savings and of Production' (1863: 103). This means that agents have to deal with securities just like they deal or would deal with their own investments: they are basically portions of capital which have to be cultivated, as sets of labour forces. In that sense, it seems to us that he remained close to the Classical paradigm and had a certain taste for labour value theory: he indeed believed that '*labor* is the real and closest measure' of the price (1863: 111) and this was closely linked to what Regnault considered as the *raison d'être* of men: labour.

'True speculation' was thus defined as 'an accumulation of labor' (1863: 6). Whereas the short-term game was based on the search for immediate profits and had no economic counterparts, the long-term speculation was associated with the development of capital and labour, and was seen as delineating a moral behaviour: 'no kind of wealth can be produced without labor; other kinds of wealth are purely chimerical' (1863: 137). In that sense, it should be clear that Regnault approached the stock market with a

harsh condemnation of the search for immediate individual interest in mind: collective interest should prevail. But how was it possible for Regnault to demonstrate these views? We have now to present his methodological choices.

### 3.2 A hymn to a statistical approach of economics

*A case of attraction for social statistics* The second part of the book can be described as a hymn to a statistical approach of the economic and social worlds. At a time when the application of statistical and probabilistic procedures in economics remained in France controversial (Ménard 1980), Regnault put forward the way in which the knowledge of society benefited from the statistical approaches initiated by Quetelet. He extensively discussed its applications to numerous issues and focused on the resulting possibility of forecasting. More precisely, statistics was for him a way to discover and to approximate deterministic laws:<sup>35</sup>

[Statistical] laws are not only the concern of material things. . . . Most importantly, they are also and as rigorously the concern of the *moral* facts, those which are precisely the less likely to belong to a stable or normal state. Births, marriages, diseases, suicides, crimes, etc. can fluctuate from year to year under the influence of accidental causes, but on a rather long period, they will appear as regular. . . . Most surprisingly, our mistakes, our distractions, our biases, and even our caprices are ruled by the law of probabilities. The human mind can appear as indiscernible. . . . Yet, the phenomena that produce it . . . appear as more regular than the physical phenomena when men are free, that is when they are not disturbed by private causes of personal interest.

(1863: 156–7)

Like Quetelet, the statistical approach Regnault had in mind was closely associated with the calculus of mean values, seen as signs of stability and order – every kind of variation from an average was considered as an error or a vice.

*Observation and time-series: Regnault's methodology* The basis of Regnault's model is thus to be found in his claim that the key for knowledge was observation:

Knowledge, or rather the intuition of future facts, . . . only results from observation, from a careful study of the previous facts: here is a truth that will never be sufficiently claimed. The future inexorably originates in the past, just like the effect constantly originates in the cause.

(1863: 140)

Consequently, the discovery of the laws that rule the stock market led him to a statistical analysis of previous observed prices.

Basically, Regnault thought that all data resulted from two components, and this idea was based on an analogy with the decomposition of movement in Physics:

When the natural laws of movement are studied in mechanics, physics, astronomy, etc., it can be seen that the movement is ruled by laws that are always the product of two simple, distinct and crucial movements. [For instance], two distinct movements rule the flight [*course*] of Earth in the space: a daily movement around itself; a yearly movement, around the Sun. In order to understand [*rendre compte*] the movement of these various bodies, it is necessary to study separately, to *decompose* the two simple movements that constitute it.

(1863: 142–3)

He thus showed that all data was including a short-term or accidental component and a long-term or constant component. The short-term components were seen as the product of causes that ‘inexorably cancel each other’ (1863: 146). By contrast, the long-term components are ‘admirably regular’ (1863: 144), and ‘the demonstration of this fact is given by the use of averages’ (1863: 150). In accordance with his deterministic views, he thus believed that some averaging procedures, associated with the law of large numbers, should be useful to reveal the long-term tendencies. Although he did not construct a formal decomposition of time-series such as those devised during the early twentieth century,<sup>36</sup> Regnault deserves a place in the history of time-series analysis: he explicitly had in mind a decomposition of time-series that will be on the forefront during the first years of the twentieth century, when early econometricians faced the problem.

With the help of these statistical procedures, Regnault zealously dealt with the discovery of deterministic laws in the field of stock market prices. His analysis was based on monthly data, relative to the highest and the lowest prices of the French 3 per cent bond, and ranging from May 1825 to October 1862.<sup>37</sup> The model of speculation was constructed in four steps.

### *3.3 The model of speculation: on constant discoveries*

*The first step: mean values and seasonal variations* The first step began with a statistical analysis of the data and was based on the determination of several mean values. For each month, during the 1825–62 period, he determined the mean value for the lowest prices and the highest prices, from which he deduced for the whole period the monthly mean values and then the general mean value (72.48F).

Then, Regnault discovered, in a quite pioneering way, the existence of seasonal effects.<sup>38</sup> He found that the lowest monthly averages occurred in June and December and that the highest monthly mean values occurred in May and November. He associated this evidence with a feature of the French 3 per

cent bond: coupon payments [*détachement du coupon*], i.e. the money that the holder of the bond gets twice a year.<sup>39</sup> He thus defined a theoretical hypothesis – the price constantly increases between the coupon payments:

Within the interval defined by the payment of two coupons, we should expect a regular increase of the prices, that would define a perfectly straight line (see the figure), because of the influence of the interest.

(1863: 153)

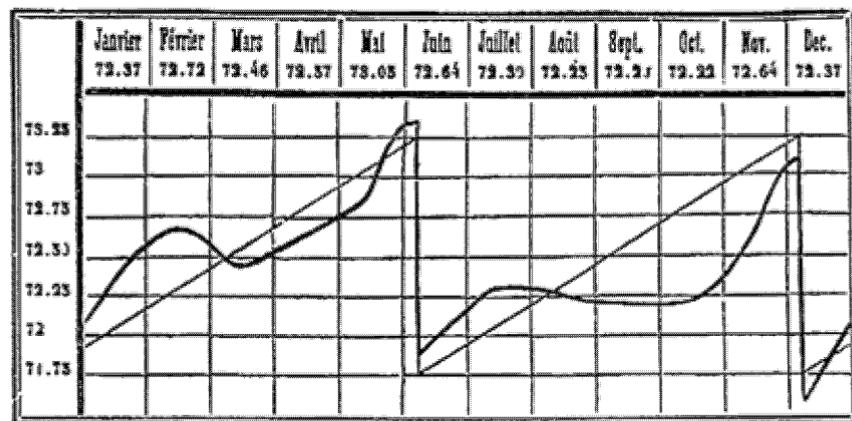
He then statistically examined the robustness of this hypothesis. Of course, he had no testing procedures available; however, he proceeded just like turn-of-the-century pioneers of econometrics did<sup>40</sup> and used a graphical approach. The observed and theoretical monthly averages are plotted on figure 1. He found that the two curves were matched and concluded that:

Among highly capricious variations, the price of the French bond is thus solely and ultimately influenced by the *constant causes*; the main one, the amount of *interest* [*le montant de l'intérêt*], is clearly identified; this apparently so weak cause will inexorably dominate the other causes.

(1863: 154)

However, he noted that the deviations of observed data from the theoretical curve could also be generated by other constant causes of a seasonal nature – such as the summer holidays.

This first step needs two comments. First, it seems perfectly clear that this kind of comparison between theoretical data and observed data is an anticipation of early econometric work. Such a graphical analysis was an



**Figure 1** The evolution of monthly mean values of the price of the French 3 per cent Bond. Observed mean values: thick line; theoretical averages: thin line (Regnault 1863: 153).

important characteristic of the French econometric tradition that rose from the 1880s – with the work of Emile Cheysson – to the 1910s – with the pioneering work of Lucien March and Marcel Lenoir.<sup>41</sup> Second, Regnault got a confirmation that the interest was a dominating cause at work.

*The second step: from the mean value to the median value* The second step can be characterized as the search for the best approximation of the mean value of the price of the French bond. Regnault believed that the previous value (72.48F) was a rather good approximation but, interestingly, he was led to consider other kinds of ‘mean values’.

His starting point is here to be found in a criticism of his data: he saw them as ‘crude’ ones, and consequently saw the 72.48F-mean value as perfectible:

It seems that the true mean value of the price can only be determined . . . on the basis of the daily *average prices* [*cours moyens*], from the highest and the lowest daily prices. These 11000 numbers would give a price around 73F . . . but this is still not the true price of the *Rente*.

(1863: 159)

He thus decided to approach the issue in a ‘roundabout manner’ (1863: 160), on the basis of monthly data. This procedure, based on the probable error, was exposed as follows:

From any mean value, the exactitude of which is dependent on [*en raison de*] the number of observations it is based on, it is always possible to calculate the error or the probable difference between this result and the true one we are looking for; the basis is the very number of observations that were used.

(1863: 160)

Considering an equal weight<sup>42</sup> for the 900 data he used, he thus computed a probable error:

$$\xi_p = \times 0.67 = 0.0223.$$

The absolute value of the probable error could then be calculated on the basis of the observed deviation, 54.15:<sup>43</sup>

$$E_p = \times 0.67 \times 54.15 = 1.20.$$

Regnault then investigated the way these probable errors were distributed around the mean value and an analysis of the observed deviations led him to think that no symmetry was prevailing:

The fluctuations in the prices of public bonds are variable, and their extraordinary deviations reveal a more marked tendency to be *inferior* to the mean value than to be superior to it. We can thus consider that the increasing state, more than the decreasing

state, is a normal one; the decreasing state is more *intense*, but less *durable*. . . In other words, the causes that generate the decline are less frequent than the causes that generate the increase, but their *strength* is superior.

(1863: 161)

From this observation, he concluded that the observed price was more frequently superior than inferior to the mean price. He then embarked on a quantification of the ‘intensity of the deviations’ (1863: 162) and found that one quarter of them occurred below the mean value. Consequently,

One has to add  $\frac{1}{4}$  of the probable difference 1.20 to the price of 72.48 in order to get the probable price of the *Rente*; this is the best approximation, given the number of data we have.

(1863: 164)

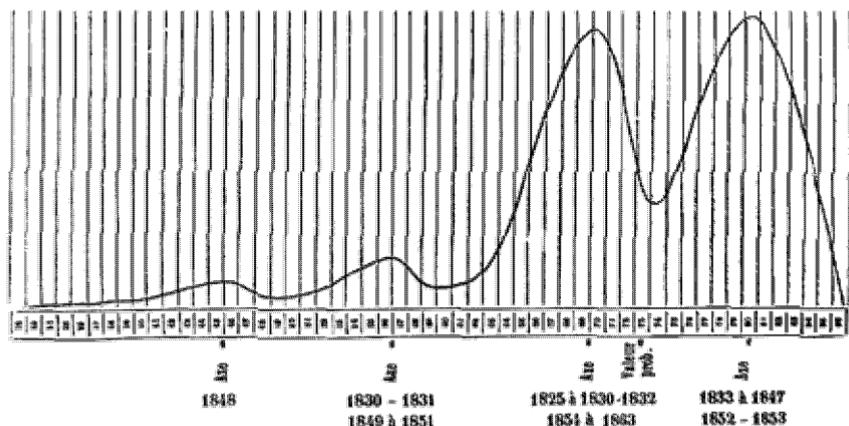
The ‘probable price’ was thus  $P_p = 72.48 + 1.20 = 73.40$ , and he concluded that ‘the variations of the *Rente* are stirring around this price’ (1863: 166).

This price that would ‘separate the number of operations in two equal parts’ (1863: 164) was in fact, in Cournot’s language (Porter 1986: 144), the median value. It should be remarked that it illustrates Regnault’s predilection for the mean value: in our case, the mean value and the median value are not equal, but he believed that in the ideal case, both would be equal.

*The third step: the structural changes* The third step can be seen as a historical analysis of the deviations from this ‘probable price’, and he discovered that other constant causes, of a ‘general’ nature, were at work: they were related to historical circumstances and led him to suggest the evidence of homogeneous periods as well as structural changes.

Once more, the analysis was based on a visual approach. Considering the monthly data for the 1825–63 period, he represented the frequency of the various prices of the *Rente* (figure 2). Regnault brought to the fore the existence of four maximum frequencies (46F, 57F, 70F and 80F) – that he labelled ‘the attraction centers’ (*centres d’attraction*) (1863: 168) – and a particular dispersion of the observed prices around these centres: their frequency ‘is decreasing, in a symmetrical way, and on the two sides’ (1863: 166).<sup>44</sup> Within the global population, he thus discovered the existence of four normally distributed sub-populations that he carefully analysed.

The first one (46F) was generated by the year 1848, characterized by political disorders. The second one (57F) mainly occurred in 1830–1 and from 1849 to 1851, and was seen as the consequence of political revolutions. Consequently, these two sub-periods were ‘abnormal’ moments of French history and interpreted as ‘deviations’ (1863: 169). The third ‘action centre’, that of 70F, occurred during the successive waves of the development of



**Figure 2** The evolution of monthly mean values of the price of the French 3 per cent Bond (Regnault 1863: 167).

industry and trade in France (in the 1830s and the 1850s). It was thus considered as a 'normal' but transitory and 'unstable state' (1863: 170). Finally, the fourth one (80F) occurred during the years 1833–47 and 1852–3 – the most prosperous years of the nineteenth century. This price was thus seen as a 'normal' one.<sup>45</sup> This historical analysis was an opportunity for Regnault to complete his results: he concluded that the 'value of the French *Rente* lies between 70 and 80F' (1863: 172). It was then possible to know if 'we should buy or sell, in a given context' (1863: 144).

*The final step: the law of attraction* The final step was that of the deduction of a deterministic law that rules long-term speculation: a kind of attraction mechanism that constantly moves the price toward the mean.

In the first part of his book, Regnault presented a law of deviations, according to which 'The deviation of the prices increases with the square root of time' and is also independent from the price itself. This law could not, of course, contribute to explain a possible relation between the observed price and a mean price. However, on the basis of the preceding steps, he put forward the existence of a second law that stipulates a relation between these deviations and the mean price. The normal distributions around the 'attraction centres' led him to think that a kind of 'force' was acting. This second law at work states that:

*The price, in all its deviations, is permanently attracted to its mean price, and this attraction is increasing with the square of its distance [La valeur, dans tous ses écarts, est sans cesse attirée vers son prix moyen, en raison directe du carré de son éloignement].*

(1863: 187)

However, it should be remarked that the demonstration of this second law was based on a unique example, that of 1848:

When the price of the *Rente* fell at 32.50F in 1848, why did it increased so rapidly and with vigor [*avec élan*] and stayed during several years at the extreme prices of 85 and 86? Because the distance to the average value was three times more important in the first case, the *Rente* . . . had a force *nine* times more important to increase than to decrease.

(1863: 188)

Apart from this example, Regnault himself dealt cautiously with this law, and confessed that he ignored ‘the measure of this force, and the very moment when it will be at work’ (1863: 188). However, he had good reasons to suppose the existence of this law: the ‘attraction law’ was indeed seen as a ‘universal law’ (1863: 168) and, in accordance with his unified conception of the world, he believed that it was also at work in the financial affairs. This is also an illustration of the fact that ‘at the apogee of classical mechanics . . . , there existed no scientific project or political utopia which did not seek a secret principle of attraction’ (Ménard 1993: 11).

This ‘law of attraction’ was undoubtedly the law Regnault was looking for. It crystallized the various constant causes at work and, consequently, aimed at proving that the short-term speculation was a mere illusion: in spite of its apparent randomness, the price of the French 3 per cent bond was dependent on a deterministic law. The short-term speculator was thus considered as ‘blind’ (1863: 198), whereas the ‘true speculator’ behaves ‘in relation with the interests; he does not focus on the current circumstances, but on the future’ (1863: 198). The end of the story was thus a moral one: the short-term speculator was favouring – and took advantage of – the existing (moral) deviations of society, but ‘the extreme things always lead individuals and people to their loss’ (1863: 210). The deterministic laws Regnault put forward had thus to induce a (moral) change in the behaviour of the economic agents: they have to learn the crucial importance of time – ‘it is necessary to WAIT’ (1863: 198), since ‘Time will always lead the price [*cours*] to their true value and will correct the deviations of speculation’ (1863: 205). The knowledge of these laws could then lead to a kind of convergence towards a state of certainty – ‘the constant feature of events invariably leads to a convergence in a more or less immediate future’ (1863: 201). Like several nineteenth-century scientists, Regnault was praising an end of history to the skies.

### Concluding comments: history as propensities

In this paper, we have tried to suggest the way a forgotten economist had developed a pioneering approach to finance. More than 30 years before

Bachelier, Regnault embarked on the construction of a model of price behaviour on the basis of new kinds of assumptions, for instance relative to information. The reader familiar with the work in finance since the 1960s will certainly be struck by the numerous innovations contained in the book – the random walk model, a measurement of volatility related to deviations, an analysis of transaction costs and of seasonal fluctuations, etc. Similarly, although we have insufficiently developed these views for space constraints, Regnault's methodology deserves special attention: the way he constantly mixed induction and deduction,<sup>46</sup> the constant focus on the test of the relations, the way these relations were also deduced from observation, the statistical procedures he used, the whole can undoubtedly be considered as signs of an econometric approach at work. In that perspective, Regnault precedes by more than 20 years the work by Cheysson that was considered by Hébert (1986) as initiating econometrics in France. More generally, Regnault's book is an illustration of the fact that econometrics rose quite early in France, in separate places and in various fields of economics, as suggested by recent work on the history of econometrics in this country.<sup>47</sup>

However, whatever the importance of these pioneering aspects, we believe that they are not historically important *per se*: we were not concerned with an intellectual genealogy, with the search for origins or with the attribution of paternity. In the paper, we rather have tried to suggest that these innovations were the product of a specific and favourable context – 'history is the sense of possibility' (Canguilhem 1992: 47). From the financial point of view, the models he constructed, as well as their underlying hypothesis, can only be understood as answers to questions related to the context of the time – of a political, moral and legal nature. Moreover, it should be remarked that his results were also dependent on the scientific views of the mid-nineteenth century. As suggested by recent histories of statistics and probability, that century cleared an important space to deterministic views, to appropriately matched statistical procedures, to the belief in the existence of an underlying order that should be discovered, and also to the belief in the existence of a convergence towards a stable state of society. Consequently, the financial as well as the econometric content of the *Calcul des Chances et Philosophie de la Bourse* are clear illustrations of the nineteenth-century paradigm. We are still ignorant of who Jules Regnault was, but there is no doubt that he is a product of that century.

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## Notes

\* A first version of the paper was presented at the 1999 HES Conference (University of North Carolina at Greensboro). We wish particularly to thank Richard Hutson for

- helpful comments. Thanks also go to Annie L. Cot, Judy L. Klein, Francisco Lou , Jean-Marc Tallon, Isabelle This and two anonymous referees.
- 1 For all  $t$  and  $T$ , the increment  $Z(t+T) - Z(t)$  is independent of the values of  $Z$  up to and including time  $t$ . This assumption is best referred to as the ‘random walk’.
- 2 Jules Regnault (1833– ? ) was an *agent de change* (broker) and worked at the Paris Bourse. He never passed the baccalauréat. To our knowledge, the *Calcul des Chances et Philosophie de la Bourse* is his only published work. However, as explained in Jovanovic (2000), we have found that this book is erroneously attributed in some dictionaries to a namesake, Jules J. Regnault, an engineer from the *Ecole des Ponts et Chaussées* who elaborated in the 1860s a modern version of the steamroller.
- 3 This filiation is analysed in Jovanovic (2001a) and is also discussed in section 3.2. As we shall explain in the paper, numerous issues at work in Regnault’s book are to be understood as financial translations of Quetelet’s results. For instance, let us mention: the predilection for mean values; the search for deterministic and long-term laws; the interpretation of variation as error, vice or imperfection; human progress as narrowing changes; etc. On Quetelet, see Porter (1986).
- 4 The mid-nineteenth century was indeed characterized in France by some resistance to the use of mathematics in economics (Ménard 1978 and 1980), and this can contribute to explaining Regnault’s predilection for a literary exposition. However, while presenting his results, he explicitly referred to ‘mathematical laws’ (see for instance 1863: 50) and mathematical formulae explicitly appear in the book when he turned the empirical test of the theoretical laws he put forward.
- 5 The French 3 per cent (or the ‘*Rente*’) was then the most important bond related to the French public debt. The coupon payment – that is the payment for an interest – occurred twice a year (in June and December). Three points can be noted here. First, approximately 450 securities were quoted in the *Bourse de Paris* and among them approximately 160 (French and foreign) bonds. Second, these securities were the concern of forward (or futures) contracts or of option contracts. Third, Bachelier’s dissertation (1900) was also devoted to the analysis of that bond.
- 6 However, the French jurisprudence implicitly recognized the futures operations on 19 January 1860 but the futures markets were only recognized on 28 March 1885. On that issue and on the juridical debates during the nineteenth century, see Tetreau (1994).
- 7 The definition of ‘*agiotage*’ changed repeatedly during the eighteenth and nineteenth centuries (see Courtois 1879). At the time of Regnault (1863: 102), the word – that he used – was related to short operations, made ‘without any resource, credit or capital, or a low amount of capital’ (translations of French books and papers are all our own). It can be noted that the French government was here paradoxically condemning a practice that it largely contributed to spread. Roussel (1859: 179–80) explained that in 1713, ‘Louis XIV signed the peace treaty of Utrecht. . . . He undertook the organization of a prestigious feast in the castle of Fontainebleau. The cost of such a feast was high: four millions francs. Such a cost could not be supported by the government, and various attempts to borrow these millions failed. They were however found through trickery. Desmarests, the *contrôleur-général* [a kind of Minister of Finance] undertook the production of 30 million banknotes . . . . It was decided to give them a fictitious value and to organize a lottery. Desmarests wrote the proposal, that lay about on his desk. It was indeed well known that his valets usually sold secrets, that they stole from their master, to speculators [*agioteurs*]. And this secret was avidly gathered and furiously exploited. The banknotes were immediately sold; it was anticipated that their value would increase, and a few days later, their price

increased from 35 to 85%. . . . The four million were collected for the *grand roi*. Once it was discovered that the lottery was based on nothing, the price of the banknotes immediately fell 66%. What a bad method for credit, based on the will of the government to consider its subjects as fools. . . . However, . . . the government immediately wanted to treat that scourge. Some *chambres de justice* [kinds of courts] were created in order to intimidate the speculators [*agioleurs*].<sup>8</sup>

- 8 The main bankers, financiers and merchants [*négociants*] of Paris signed numerous petitions to the Senate. Their number increased in the 1860s.
- 9 Selling short means that you sell an asset that you do not already own.
- 10 See Frèrejouan du Saint (1893) and Boboef (1864) on this issue.
- 11 In Paris, legal operations were exclusively made through the intermediary of these brokers. We can note that in a sense, some of the moral arguments hark back to the medieval distinction between the acceptability of games made by conveying goods from one place to another and the declared illegitimacy of games entirely attributable to the time difference in value. This parallel is also clear from the various steps that led to the elaboration of the *Code Napoléonien*: as noted by Vercamer (1903: 6), the condemnation of speculation was based on arguments developed by Thomas Aquinas.
- 12 The *Journal des Economistes* – the most influential economic journal of the time – recurrently published debates among the members of the Société d'Economie Politique (see for instance the *Journal des Economistes* 1857, 47: 308–14).
- 13 The jurists were generally ignoring the way the stock market was operating and this explains the role played by the economists. The former were often roughly claiming that the speculation could be assimilated to a game of chance and were believing that both should be forbidden according to similar principles (see Frèrejouan du Saint 1893). Note also that the legislation as well as the debates of the time were not the concern of the structure and the organization of the markets but of the financial operations that were realized on these markets.
- 14 Alphonse Courtois – the father of the former Alphonse Courtois – published in 1852 in the *Journal des Economistes* a study that aimed at defending speculation upon arguments based on its economic need. Note that the 1850s were characterized by a rapid development of the stock market, illustrated by the number of quoted enterprises and the amount of capitalization. The ratio capitalization/GNP was 1.9 in 1820, 6.7 in 1850 and 17.5 in 1855 (Arbulu 1998).
- 15 This illustrates the opposition between the traditional economists of the time, who used literary language exclusively, and a younger and more pragmatic school that turned to 'new methods', such as statistics and graphs.
- 16 Lefèvre was the personal secretary of James de Rothschild (an influent banker); he was also a collaborator of the *Journal des Actuaires Français*, a banker and the editor of a journal specialized in financial issues. As early as 1870, he devised a graphical method to represent the gain function of the speculator (see Zylberberg 1990; Jovanovic 2001b). This method was well known by the *agents de change* (Lefèvre 1874) and was discussed in his own journal in 1870. He constantly claimed the need for financial lectures and he became professor at the Institut Polytechnique.
- 17 However, he recognized that the separation between short-term and long-term speculation was merely a theoretical one, just like the one between right and wrong (1863: 103–4).
- 18 See Hacking (1990) on nineteenth-century determinism.
- 19 This distinction was also at work in Quetelet's work. See Porter (1986: 108).
- 20 According to Armatte, the contemporary meaning of the mean value ('*la moyenne*')

appeared in the French language at the very beginning of the seventeenth century and came from the Latin words ‘*mediocris*’ and ‘*mediocitas*’. Both words were not used in a pejorative sense but meant an intermediate place between two extremes. ‘Thomas Aquinas is the theologian of this mediocrity that becomes the place of all the virtues. Then the just middle became the model of religious and non-religious morals’ (1995, ch. 5: 24), in the sense that it became a model of justice and rational decision.

- 21 See for instance Cohen (1993b), Israel (1996), Le Gall (1999), Mirowski (1989, 1994) and Schabas (1990).
- 22 See Cohen (1993a) and Ménard (1993) for such an approach.
- 23 See also Israel (2000). See Le Gall (2001a) for a historical analysis of econometric modelling at the light of Israel’s framework.
- 24 By model, we mean ‘a representation . . . of an existing object or an object to be created’ (Israel 1996: 17–8). Regnault’s models were *mathematical* models, although generally exposed in a literary style (see footnote 4). Note that, following Morrison and Morgan (1999), we see models as partially independent from theories.
- 25 Such moral issues stimulated the emergence of probability theory. See Coumet (1970) and Hacking (1975).
- 26 For a long time, financial ‘theory’ was only relative to the finances of the government, actuarial practice and legal problems linked to financial operations.
- 27 See Jovanovic (2000) on the relation between Regnault and Bachelier.
- 28 In the opposite case, the speculator has to pay an amount of money (the ‘*report*’).
- 29 If a large number of individuals are at an identical distance from a building and have to evaluate its height, their own estimations will be different. However, they will not be randomly distributed: the ‘law of chance’ rules them. Regnault indeed explains that ‘the values given by each individual will not be randomly ruled, or without any order; they will be distributed [*se grouperont*] according to a certain law, and in the most symmetrical way around [*des deux côtés*] the mean value; if we divide in equal parts the distance from this value to the extreme values, the numerical value of each group would constantly and progressively decrease with the distance from the mean value’ (1863: 25; our italics). At that time, this example was a classical one (see Feldman *et al.* 1991), and Porter (1986: 107) also shows the way it was also used by Quételet. Note that the term ‘normal law’ was only introduced by Karl Pearson in 1893. Although this law was never explicitly put forward by Regnault, his diagrams as well as numerous parts of his book led us thinking that his analysis was based on the Gaussian law (see for instance 1863: 167), which was a means to bring deviations within the domain of order.
- 30 The mean deviation was the mean value of the extreme prices during a certain period. Note that he also calculated a probable deviation (i.e. the median deviation). The difference between these two deviations reveals the instability of the market. If the prices were affected by no large variation, there would be no difference between the mean and the probable deviations. He saw this difference as a measure of the ‘morality’ of the market: a mean deviation superior to a probable deviation indicates that the short-term speculation dominates and the market can then be considered as volatile. In Regnault’s own words: ‘We can note the remarkable ratio that links the probable deviation to the mean deviation, about 2/3, a ratio that we frequently find, for instance the one which links the probable life to the mean life. Moreover, this ratio is not constant, it is a measure of the regularity of the movement of prices [*cours*], and depending on whether this movement is acting on more or less violent shocks . . . , the probable deviation gets closer to the mean deviation; they even could

be identical if the movement of prices [*cours*] was perfectly regular and continuous. And just like the tendency for the probable life to get closer to the mean life is an sign [*indice*] of the progress of the well-being [*bien-être*] and of the civilization in a nation, the ratio of prices deviations, which tends to get closer to unity . . . , is an exact measure of the morality of speculation' (1863: 53). See also the conclusion of Regnault's book.

- 31 A similar law was at work in Quetelet (see Jovanovic 2001a).
- 32 Such a test was also at work in Bachelier's dissertation (see Jovanovic and Le Gall 2001b).
- 33 See also Regnault (1863: 77–95).
- 34 'The reason why the prices on the stock market are significant is the fact that, at a given time and a given price [*cours*], there will be a buyer who will be able to get a security in exchange for his capital, there will be a seller who will get the payment of capital in exchange for his security. . . . The rise will then not be a simple phenomenon, fruitless and without any result, because its direct effect is the appreciation [*une plus-value*] of the other existing securities; by contrast, the decline will result in their depreciation. It is true that the rise has contributed neither to the creation of any security, nor to add a centime to social assets. Its result is the creation of a moral value [*valeur morale*], that we label trust or creditworthy [*crédit*] and that can perfectly be measured [*susceptible d'une appréciation chiffrée*] just like concrete and material wealth' (1863: 109).
- 35 This relation between statistics and determinism is analysed in Hacking (1990) and Porter (1986).
- 36 We have for instance in mind the methods devised by Lucien March (1905), which were based on the estimation of a trending factor (a moving average); the deviations from the observed data were seen as oscillatory short-term components (see Jovanovic and Le Gall 2001a; Klein 1997).
- 37 The data are presented in Regnault (1863: 148–9).
- 38 In a paper read to the British Association for the Advancement of Science in 1862, Stanley Jevons took a similar approach to seasonal fluctuations. This paper is reproduced and analysed in Hendry and Morgan (1995). Note moreover that seasonal effects were also put forward by Quetelet.
- 39 More precisely, he remarked that the maximum price of the French bond precedes the days of its coupon payment (6 June and 6 December), whereas its minimum price happens on the following day.
- 40 See Morgan (1997).
- 41 See Ekelund and Hébert (1999), Hébert (1986), Le Gall (1997) and Zylberberg (1990) on Cheysson's econometric work on demand. See Jovanovic and Le Gall (2001a) on March's views on statistics and econometrics. See Chaigneau and Le Gall (1998), Hendry and Morgan (1995) and Morgan (1990) on Lenoir's econometric work on demand and supply. See also Klein (1997) for a historical analysis of the graphical approach.
- 42 This choice was based on the conviction that the data were the product of a homogeneous trend.
- 43 54.15 is the difference between the highest monthly mean value (86.65F in July 1840) and the lowest one (32.50F in April 1848).
- 44 This was, once more, a confirmation that the world was normally ruled: this result 'is in accordance with the universal laws that rule the world' (1863: 168).
- 45 Regnault claimed that a well-organized society would only admit a unique 'gravity center'. One can also note that he had an ideal society in mind, and history was for him that of a possible convergence toward it, *à la Cournot*.

46 See Jovanovic (2000) for an analysis of that issue.

47 See for instance Breton (1992), Chaigneau and Le Gall (1998), Desrosières (1993), Jovanovic and Le Gall (2001a), Le Gall (1997 and 2001b).

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## Abstract

We analyse the work of a neglected French economist, Jules Regnault, whose *Calcul des Chances et Philosophie de la Bourse* (1863) laid the basis of modern stochastic models of price behaviour and contains an anticipation of econometrics. At a time when short-term speculation was denounced as immoral, he approached this question ‘scientifically’ and constructed two models. The first one was relative to short-term speculation and took the shape of a random walk – a model used by Bachelier (1900). The second one deals with long-term speculation and aims at evaluating the mean value of the French 3 per cent bond.

## Keywords

Random walk, history of economic thought, econometrics, financial theory