

Towards a transdisciplinary econophysics

Franck Jovanovic

TELUQ – UQAM (Université du Québec à Montréal), Canada

Christophe Schinckus

University of Leicester, UK

Abstract

This paper deals with the disciplinary dimension of a very new field called econophysics and shows that despite the fact that econophysics is regularly described as an interdisciplinary approach, it is in fact a multidisciplinary field. Beyond this observation, we note that recent developments suggest that econophysics could evolve towards a more integrated field. We have therefore taken a prospective approach by analyzing how this field could become transdisciplinary. We show that a common scheme is attainable and we investigate the possibilities of transdisciplinary econophysics.

The contribution of this article is twofold: on the one hand it clarifies the epistemological status of econophysics, and on the other, it studies the recent evolution of econophysics and shows how this field could evolve to become transdisciplinary.

This article is part of a more general project that aims to build a transdisciplinary econophysics, or “trans-econophysics.” Trans-econophysics will make it possible to revisit the theoretical foundations of financial economics, rethink radical (Knightian) uncertainty, and develop new models and theories better suited to the management of financial risks and financial markets. Trans-econophysics will contribute to the building of a common theoretical framework by giving financial economists accessible econophysics models and results. Consequently it will create an opportunity for transforming financial economics.

I. Introduction

This paper deals with the disciplinary dimension of a very new field called econophysics. This field, created outside financial economics by physicists from statistical physics, studies economic phenomena, and more specifically financial

markets, using various models and concepts imported from condensed matter and statistical physics. In other words, econophysics is characterized by the application of models from particle physics (which is a subfield of statistical physics) that use stable Lévy processes to financial markets¹. This recent approach is often presented as an “in-between” field – a field between physics and economics, and more particularly financial economics (Keen 2003, Gingras and Schinckus 2012). Multidisciplinarity, interdisciplinarity and transdisciplinarity refer to different levels of integration of several disciplines. Analyzing these types of integration, this article shows that despite the fact that econophysics is regularly described as an interdisciplinary approach, it is in fact a multidisciplinary field. More specifically, as we explain, econophysics is not properly speaking an “in-between” field. Beyond this observation, we note that recent developments suggest that econophysics could evolve towards a more integrated field. Thus we have taken a prospective approach, analyzing ways in which the field could become transdisciplinary. From this perspective, we show that a common scheme is attainable and we investigate the possibilities of transdisciplinary econophysics.

The contribution of this article is twofold: on the one hand it clarifies the epistemological status of econophysics, and on the other it studies the recent evolution of econophysics and shows how the field could evolve to become transdisciplinary.

¹ On the history of econophysics see Jovanovic and Schinckus (2013), Roehner (2002) or Daniel and Sornette (2010).

This article is divided into three parts. The first part briefly presents econophysics and its main links with economics. The second part analyzes the disciplinary dimension of econophysics and shows that econophysics is a multidisciplinary field. The third part analyzes what “trans-econophysics” would be.

II. Econophysics, a new field of research

For over a decade, a considerable number of physicists have been applying concepts from physics to study economic phenomena. The term “econophysics” is now generally used to describe this work. According to Kutner and Grech (2008), econophysics as a field of research dates back to 1991 when Mantegna published a paper about the use of Lévy processes in finance. However, Jovanovic and Schinckus (2013) trace the roots of the basic ideas of econophysics to Benoît Mandelbrot (1963, 1965), who saw an analogy between the evolution of financial markets and the phenomenon of turbulence. Despite these and some papers on pure Lévy processes in finance written in the mid-1960s (Fama 1965, Samuelson 1965), this statistical approach was not pursued at that time, essentially because of its incompatibility with the theoretical framework used by economists (Jovanovic, *et al.* 2013)².

² The basic incompatibility stems from the indeterminacy of variance in Lévy processes. From a financial perspective, this would imply indeterminacy of the measurement of risk. We evoke this point in the last part of this article.

Econophysics, as a specific label and conceptual practice, was first coined by the physicist H. Eugene Stanley in 1996 in a paper published in *Physica A* (Stanley, Afanasyev, *et al.* 1996). As the name suggests, econophysics presents itself as a hybrid discipline which can be defined in methodological terms as “a quantitative approach using ideas, models, conceptual and computational methods of statistical physics” applied to economic phenomena, and especially financial phenomena (Burda, Jurkiewicz, *et al.* 2003, 1).

The influence of physics on financial economics is nothing new³. A number of writers have studied the “physical attraction” exerted by economists on physics (Mirowski 1989, Ingrao and Giorgio 1990, Schabas 1990). But as McCauley (2004) points out, in spite of these theoretical and historical links between physics and financial economics, econophysics represents a fundamentally new approach that differs from preceding influences. Its practitioners are not economists taking their inspiration from physics to develop their own discipline, as has been seen repeatedly in the history of economics. This time, it is physicists that are going beyond the boundaries of their discipline, studying various problems thrown up by social sciences in the light of their methods and models.

³ One of the first authors to bring physics closer to the financial domain was Jules Regnault in the second half of the 19th century (Jovanovic and Le Gall 2001, Jovanovic 2006). In the 20th century, a number of physics concepts played a part in the development of modern financial theory. The best known application of physics to finance is the application of the heat-diffusion formula (Bachelier, Black and Scholes) and a number of studies implicitly or explicitly referred to a concept from the field of physics: Brownian motion (Jovanovic 2009).

Using standard tools of statistical mechanics such as microscopic models, the Ising model or scaling laws, econophysicists attempt to explain how “emergence” appears at the macro-level of complex economic systems⁴. Epistemologically, econophysics is founded on a belief in the universality of some general statistical properties that reappear across many and diverse phenomena (McCauley 2004). This statistical universality can be characterized by scaling laws that are considered to be at the heart of econophysics⁵ – (Bouchaud 2002), Stanley and Gabaix (2008, 288). These scaling laws can take a variety of forms. According to most econophysicists (Mandelbrot and Hudson 2004, McCauley 2004) complex economic systems⁶ obey a specific kind of invariance that can be characterized by power-law distributions of the form $P(X > x) = x^{-\alpha}$, where $p(x)$ is the probability of there being an event of magnitude x and the scaling exponent α is a constant whose value is set either by the empirically observed behaviour of the system, by a theory or by simulations⁷. Econophysics' distinctive feature is that it gives a very particular

⁴ Some authors (McCauley 2004, Israel 2005) argue that the idea of “emergence” is empty and should be replaced by the physics-based concept of invariance, Rosser (2008b) showed that the distinction between the two is irrelevant and results from the old methodological struggle between the continuous and the discrete. See Rosser (2008b) for a very good introduction to this point.

⁵ These scaling laws can then be viewed as a macro result of the behaviour of a large number of interacting components from lower levels. As Rickles (2007, 7) explains, “The idea is that in statistical physics, systems that consist of a large number of interacting parts often are found to obey ‘universal laws’ - laws independent causally of microscopic details and dependent on just a few macroscopic parameters”.

⁶ Even if power law distributions are also used to characterize many phenomena in social sciences (such the ranking of firm size (Stanley, *et al.* 1996); the income distribution of companies (Okuyama, Takayasu, *et al.* 1999); fluctuations in finance (Mandelbrot 1997)), these laws are often replaced by log-normal laws in which the variance parameter is not infinite.

⁷ Pareto was the first to investigate, in his *Cours d'Economie Politique* (1897) the statistical character of the wealth of individuals by modelling them using power laws.

form to these power laws by using stable Lévy stable processes for which the exponent α lies between 0 and 2⁸.

The development of this new discipline raises questions about its differences and its potential contribution to financial economics. How could physicists who explicitly reject financial theories contribute to a better understanding of financial phenomena? How could econophysics contribute to financial economics? How might it be possible to develop knowledge common to financial economists and physicists? These questions relate to the disciplinary dimension of econophysics.

III. The disciplinary dimension of econophysics

In the specialized literature, econophysics is often described as an interdisciplinary field: “[econophysics is an] interdisciplinary research field applying methods of statistical physics to problems in economics and finance” (Săvoiu and Iorga–Simăn 2008, 33). Although the definition of econophysics varies from author to author, most econophysicists seem to share the idea that their approach is really an interdisciplinary field. For instance, Aste and Di Matteo have stated that “Econophysics is an interdisciplinary field which applies the methods of statistical physics, nonlinear dynamics, and network theory to macro-micro/economic modeling, to financial market analysis and social problems” (2009, 3). Similar definitions can be found in the work of many authors, for

⁸ If $\alpha > 2$, then the Lévy process is no longer stable.

example, Mantegna and Stanley (1999), Gligor and Ignat (2001), Vasconcelos (2004), Carbonne et al. (2007), and Daniel and Sornette (2010). However, in this part, after defining the terms multi-, inter- and transdisciplinary, we demonstrate that this new field is a multidisciplinary one.

III.1. Multi-, inter-, and transdisciplinary

The association between disciplines is a relatively new phenomenon. As Max-Neef (2005, 6) emphasizes, this development “has been significant for the maintenance of disciplinary autonomy, for the competition of research funds and for the consolidation of academic prestige.” While disciplinarity refers to a mono-discipline describing a specialized scientific field, notions such as multidisciplinary (or pluridisciplinarity), interdisciplinarity, and transdisciplinarity imply a variety of disciplines. This section clarifies the concepts of multi-, inter-, trans-disciplinary, with the proviso that in this article we focus only on an epistemological perspective – see Max-Neef (2005) or Choi and Pak (2006) for a more sociological approach.

Multidisciplinarity implies several disciplines and provides knowledge that stays within the boundaries of the fields involved. More precisely, several disciplines are in association for the purpose of analyzing a common object with their own theories, models and concepts. Klein (1990, 110) explained that multidisciplinary is “a process for providing a juxtaposition of disciplines that is

additive, not integrative; the disciplinary perspectives are not changed, only contrasted.” In other words, each field’s knowledge provides a different perspective on a particular problem or issue. In this perspective, scientists have separate but related roles (since they study the same subject of research) but have separate methodologies (Flinterman, Teclemariam-Mesbah, *et al.* 2001), and the aim of a multidisciplinary project is often “instrumental” (i.e. used to solve a specific problem: (Whitfield K 2004)). Multidisciplinarity can be represented using the following graphs proposed by Max-Neef (2005, 7):

Multidisciplinarity (No cooperation) :

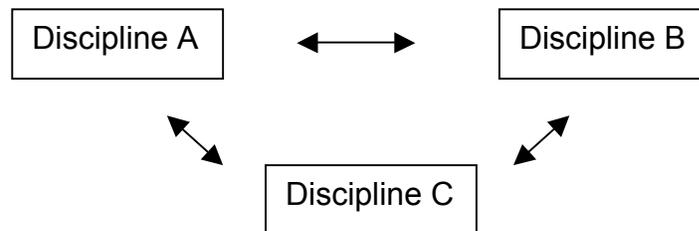


Multidisciplinarity characterizes, for example, the case of a person who has studied several disciplines (but who never makes connections between them) or the case of an academic department in which several experts work but without connections. In terms of knowledge, an example might be an event such as World War II, which is studied from a strictly historical or economic point of view without cooperation between each expert.

Pluridisciplinarity is very close to multidisciplinarity, except that there is an attempt at cooperation between the disciplines involved (without coordination). This kind of process “normally happens between compatible areas of knowledge”

(Max-Neef 2005, 5). In a sense, a pluridisciplinary project means research in which all experts take into account the different viewpoints involved.

Pluridisciplinarity (Cooperation without coordination):

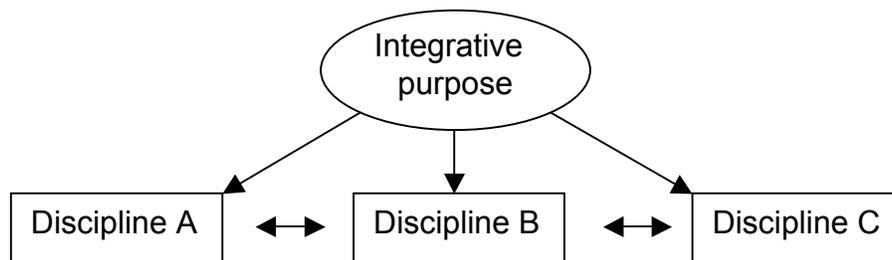


Pluridisciplinarity involves the use of a combination of disciplines in the study of a specific area of knowledge. The approach of each discipline contributes to a better understanding of the subject under study. The disciplines involved often cover compatible areas of knowledge, such as physics and chemistry, or sociology and politics. Among the many examples that could be cited are all phenomena related to heat, an understanding of which contributed to the development of the disciplines involved: physics and chemistry.

Interdisciplinarity is a very different approach because it refers to “joint, coordinated and continuously integrated research done by experts with a different disciplinary background, working together and producing joint reports and papers” (Grossman 1979, 54). An interdisciplinary team aspires to a more profound level of collaboration than a multidisciplinary team: “different

backgrounds combining their knowledge mutually complete different levels of planned care” (Bernard-Bonnin, Stachenko, *et al.* 1995, 35). In this perspective, participants have common roles and they try to arrive at integration and synthesis of the disciplines involved by developing a common methodology, models and theories ((NSERC) 2004). Disciplinary knowledge, concepts and tools of investigation are considered and combined in such way that the resulting understanding is greater than the sum of its disciplinary parts. As Max-Neef (2005, 7) explained it, interdisciplinarity often has an integrative purpose:

Interdisciplinarity (Cooperation and coordination):

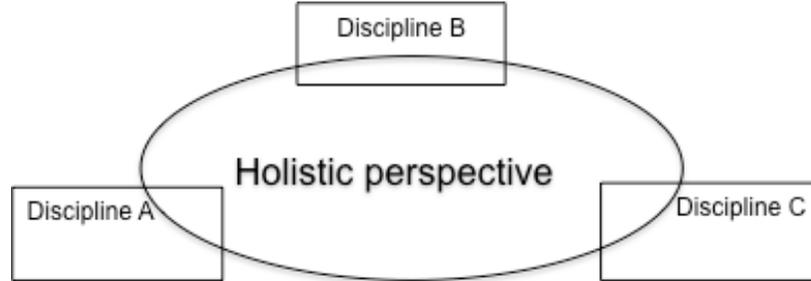


This integrative purpose can be pragmatic or normative. Medicine, for example, is based on empirical facts coming from biology, chemistry, psychology and even physics (for medical imagery). Medicine can thus be seen as an interdisciplinary field based on pragmatic purpose because all analyses involving several disciplines are focused on a pragmatic result (diagnosis or treatment). Engineering can also be considered as a pragmatic interdisciplinary field. Max-Neef (2005, 7) explained that politics, for example, is more of a normative interdisciplinary field since it indirectly touches on economics and sociology.

Transdisciplinarity describes the most integrative collaboration between disciplines. “Transdisciplinary projects are those in which researchers from different fields not only work closely together on a common problem over an extended period but also create a shared conceptual model of the problem that integrates and transcends each of their separate disciplinary perspectives” (Rosenfield 1992, 55). All participants then have common roles and try to offer a holistic scheme that subordinate disciplines. Transdisciplinary research is “concerned with the unity of intellectual frameworks beyond the disciplinary perspectives” (Stember 1998, 341). This kind of research involves issues in which each disciplinary knowledge involved takes into account the other disciplinary frameworks by adapting to them. In a transdisciplinary perspective, disciplines must be looked on as necessarily complementary in order to better understand the complexity of reality. Rather than forming a new discipline or a supra-discipline, transdisciplinarity implies a more systemic and holistic way of thinking about the world. The objective is to arrive at knowledge that transcends disciplines in order to create a common theoretical framework. In other words, while multidisciplinary and interdisciplinarity are merely continuous extensions of disciplinary, transdisciplinary implies unity. We can represent transdisciplinarity as follows:

Transdisciplinarity

(Cooperation with integrative collaboration):



It is not so easy to find examples of transdisciplinary research because it implies a very integrative perspective. The ecology of today can be looked on as a transdisciplinary discipline. Max-Neef (2005) explained that *growth* and *environment* were frequently identified as opposites in conventional economics because they were mainly based on anthropocentric reasoning. By taking into account different fields (economics, demography, biophysics etc), a more biocentric ecology has been recently developed. This ecology has proposed a new framework in order to solve the traditional opposition between *environment* and *growth*. In this perspective, these opposite concepts can now be seen as complementary in a unified *development*.

III.2. Econophysics: a multidisciplinary field

Although econophysics and financial economics share the same topics (mainly the analysis of stock-price variations), they differ in the mathematics they use and the constraints they have to face. Econophysics' distinctive feature is the use of stable Lévy processes for modelling stock-price variations, while financial

economics is based on Gaussian framework (the last part will explain this difference in greater detail). Concerning constraints, econophysicists focus on the application of mathematical models to empirical data. The fit between model and empirical data is the only constraint that statistical physicists have to face. By opposition, financial economists have to face to a second constraint: the compatibility between models and the theoretical framework, which mainly comes from economics⁹.

To be an interdisciplinary field, econophysics should provide an integration and a synthesis of economics and physics by developing a common methodology, models and theories. However, up to now, all models developed by econophysicists have stayed within the boundaries of statistical physics. Indeed, econophysicists try to explain economic phenomena only with theoretical tools, models and methods derived from physics. Chatterjee et al. explain:

“The main force behind this outflow is not so much that physicists have lost interest in physics, but the realization that there are incredibly interesting complex phenomena taking place in other disciplines which seem now within the reach of the powerful theoretical tools which have been successful in physics” (Chatterjee, Yarlagadda, *et al.* 2005).

Econophysicists are not attempting to develop common models or theories by making a synthesis with models or theories from economics. They are applying physics concepts and models as they exist today. In fact, they claim to be

⁹ See Jovanovic (2010) for an illustration with efficient market theory.

seeking to ignore, even deny, economics in an endeavour to replace the theoretical framework that currently dominates it with a new framework derived directly from statistical physics¹⁰. In other words, econophysics aims to provide a new perspective on stock-price variations – a traditional subject of financial economics – using tools and models from physics.

In addition, most of the models used by econophysicists are not used by financial economists because they are incompatible with financial economics' theoretical framework. Jovanovic and Schinckus (2013) point out that Mandelbrot (1963, 1965) and Fama (1963, 1965) initiated a theoretical movement by using stable Lévy processes to describe financial markets. Fama (1965) also proposed a stable Lévy version of the portfolio theory. However, stable Lévy processes were soon put aside in financial economics because the variance of stable Lévy processes does not tend towards a fixed value: the variation is said to be *infinite*¹¹ (Jovanovic and Schinckus 2010a, 2013).

The infinite-variance hypothesis is meaningless in the framework of financial economics and one of the major theoretical constraints for financial economists. Variance and the expected mean are the two main variables for their theoretical

¹⁰ During past decades, many physics models were employed in economics, but these models were mainly used for their mathematical description of physical phenomena. Progressively, these imported models have been integrated in the mainstream (see Black and Scholes model, for example). This trend is not observed with econophysics, in which economic phenomena are explained in terms of molecular mechanisms or complex interactions. From this point of view, econophysicists do not try to connect their works with the pre-existing economic theory. For an epistemological analysis of this attitude, see Rosser (2008b).

¹¹ The adjective “indeterminate” would be more accurately employed, but the literature uses “infinite”.

interpretations. In the 1960s, the period in which financial economics was constituted as a scientific discipline, the relationship between risk and return was taken from Markowitz' work (1952, 1959). Markowitz associated risk with variance and return with the mean. Today, the whole of financial economics is built on this association. In this perspective, if variance were infinite (as it is in a Lévy process), it would be impossible to understand the notion of risk as Markowitz had defined it and, consequently, as it is understood by financial economists. Physics models were thus incompatible with the framework of financial economics at the outset.

The gap between the two disciplines is clearly set out in the literature of econophysics, which often criticizes financial economics by emphasizing, for instance, the “superficially appealing” nature of its concepts or by describing the field as a “tapestry of beliefs” (Keen 2003, 108)¹². McCauley, one of the biggest names in econophysics, has stated that “econophysicists are safer to ignore the lessons taught in standard economics texts” (McCauley 2006, 602)¹³. Econophysics has generated methodological debates between economists and physicists despite this radical position. However, these debates have not led to any change in the disciplinary perspectives. We merely observe the development of a literature emphasizing methodological contrasts between econophysics and economics (Keen 2003, Gallegati, Keen, *et al.* 2006, Schinckus 2010). Today,

¹² With Rosser (2006, 2008a), Keen is one of the rare breed of economists who have engaged with econophysicists.

¹³ McCauley (2006, 17) did not hesitate to compare financial theory to cartoons: “the multitude of graphs presented without are not better than cartoons because they are not based on real empirical data only on falsified neoclassical expectations.”

econophysics is an autonomous field (Jovanovic and Schinckus 2010b, Gingras, *et al.* 2012), and is not an integrative and collaborative approach between econophysicists and economists. And because there is no integration or synthesis of physics and economics through the development of a common methodology, models and theories, it is evident that we cannot consider econophysics as an interdisciplinary field, and thus not as a transdisciplinary field either.

Despite this conclusion, a remark from Rosser (2003, 1) is worthy of note: “It can be argued that ‘transdisciplinary’ might be a better label for econophysics. [...] ‘Multidisciplinary’ suggests distinct disciplines discussing as with an economist and a physicist talking to each other. ‘Interdisciplinary’ suggests a narrow specialty created out of elements of each separate discipline, such as a ‘water economist’ ” who knows some hydrology and some economics. However, “transdisciplinary” suggests a deeper synthesis of approaches and ideas from the disciplines involved. Although econophysics today must rather be regarded as a multidisciplinary approach, considering the recent development of this field, the very interesting question raised by Rosser (2003) must be asked: would a transdisciplinary econophysics be possible? In the next section, we will explore this question and the possibilities of such an evolution of the field.

IV. Towards a “transeconophysics”

A transdisciplinary econophysics would imply a more integrative approach in which econophysicists and economists would share a common conceptual scheme that transcends both disciplines. This “integrative dimension” refers to two kinds of integration: on the one hand, a methodological integration to produce a common conceptual framework and, on the other hand, a sociological integration – meaning that theorists from the disciplines involved go beyond their cultural differences in order to work together in a common project. The sociological integration is a matter of “inter-professionality” related to the standardization of knowledge “through the background” (D'Amour and Oandasan 2005) while the methodological integration refers to the knowledge itself. In this part, we focus on the methodological issue of the transdisciplinarity of econophysics¹⁴ for analyzing a possible common scheme between econophysics and financial economics despite the current situation, and consequently the possibility for econophysics to become a transdisciplinary field.

IV.1. What transdisciplinary econophysics would be

As explained in part II, transdisciplinarity refers to knowledge that transcends disciplines in order to create a common conceptual scheme. Morin (1994) explains that “the big problem is to find the difficult path of the inter-relationship [*l'entre-articulation*] between sciences that have not only their own language, but

¹⁴ See Gingras and Schinckus (2012) for a more sociological analysis of the emergence of econophysics.

basic concepts that cannot move from one language to another.” Although a transdisciplinary project requires that disciplines share common features, and in particular a common conceptual scheme, the problem of language (concordances) must be also considered. Klein (1994) explains “A ‘pidgin’ is an interim [i.e. an in-between] tongue [i.e. language], based [on] partial agreement on the meaning of shared terms [...]. Transdisciplinarity [...] will begin with a pidgin, with interim [i.e. in-between] agreement on basic concepts and their meanings.” Such an in-between language should be articulated with an in-between conceptual scheme that would be progressively developed.

Regarding the history of econophysics, Jovanovic and Schinckus (2013) explain that the field was created because some continuities already existed between statistical physics and financial economics due to the fact that Gaussian processes are a particular case of Lévy processes. However, these continuities did not imply an in-between language or conceptual scheme. Following the analysis of Morin and Klein, a transdisciplinary econophysics must take into account the constraints that both original disciplines have to face: for statistical physicists, the fit between model and empirical data; for financial economists, the fit between model and empirical data on one hand, and the compatibility between model and theoretical framework on the other hand (see part III). Thus, this in-between language implying a common conceptual scheme must result from a double movement: models from physics must incorporate the theoretical framework from financial economics and, at the same time, theories and

concepts from financial economics must be modified so that they encompass for richer models from physics.

This double movement is a necessary step towards a more integrative econophysics. Indeed, the creation of a transdisciplinary field requires an adaptation of each field involved in order to transcend each of their separate disciplinary perspectives. This adaptation implies the integration of theoretical constraints observed in each discipline in such a way that the new shared conceptual framework will make sense in each discipline.

This double movement can in fact be seen in the creation of econophysics. While there were originally no tools that could make physics models (such as those used in econophysics) compatible with the theoretical framework of financial economics (see part III), we observe the emergence of this double movement in the 1990s. As we will explain in the next section, recent works suggest that a common framework between econophysicists and financial economists is possible, and consequently, a trans-econophysics seems conceivable.

IV.2. A trans-econophysics project

From Gaussian framework to exponentially truncated Lévy framework

Financial economists mainly use the Gaussian framework in order to characterize financial uncertainty. For econophysicists, the Gaussian framework is the first step of describing uncertainty in science. This first step can be generalized through an uncertainty “without normality” (Mandelbrot 1997, 66) based on the use of Lévy processes to describe complex phenomena. In this perspective, a condition for developing a theoretical bridge between econophysics and financial economics is a common framework in which the traditional Gaussian approach used in finance would make sense with the mathematical models used in econophysics.

The first step that could lead to the creation of transeconophysics has been taken with the use of truncated Lévy distribution, which made it possible to solve the infinite-variance problem. Indeed, this statistical solution opens the possibility of using stable Lévy processes to describe the evolution of financial prices¹⁵, and consequently, the possibility of creating a common framework.

Truncating a Lévy distribution consists in normalizing it using a particular function so that its variance is finite. In this perspective, stable Lévy distributions are used with the specific condition that there is a cut-off length for the price variations above which the distribution function is set to zero in the simplest case (Mariani and Liu 2007) or decreases exponentially (Gupta and Campanha 2002, Gunaratne and McCauley 2005). These functions are chosen in order to obtain

¹⁵ Note that truncated stable Lévy processes can be seen as the statistical solution to the problem of infinite variance emphasized by Mandelbrot (1963) and Fama (1965).

the best fit with the empirical data (Mariani, *et al.* 2007) or are derived from models like percolation theory (Gupta, *et al.* 2002) or the generalized Focker Plank equation (Gunaratne and McCauley 2002). Generally speaking, the probability distribution of a truncated Lévy distribution can be defined as followed,

$$P(x) = \begin{cases} N(0, \sigma) & |x| > l \\ k L(x)_{(\alpha, \beta, \gamma, \delta)} & |x| \leq l \end{cases}$$

Where $L(x)_{(\alpha, \beta, \gamma, \delta)}$ is a symmetrical Lévy stable distribution with an index α ($0 < \alpha \leq 2$), a scale factor $\gamma > 0$ and $\beta = 0$ since the distribution is symmetrical. δ is a scale factor which is positive, k is a normalizing constant and l is the cut-off length. Usually, the standard Lévy distribution is abruptly cut to zero at a cut-off point (Mantegna and Stanley 1994). In other words, the probability of taking a step is abruptly cut to zero at a certain critical step size.

With truncated Lévy distribution, physicists can have finite variance, and can apply models that use this distribution to financial economics' theoretical framework. This truncation of Lévy processes is a very important step towards a transdisciplinary econophysics because it allows the use of power laws (with $0 < \alpha < 2$) in financial economics. That is exactly the kind of statistical solution that Fama (1965) wanted to develop. In other words, truncated Lévy processes could make it possible to achieve what financial economists were trying to do at the very beginnings of the discipline. This development of truncated stable Levy

processes offering finite variance was the first step towards a more integrative econophysics. However, such processes are not fully compatible with the theoretical framework of financial economics as it exists today, and particularly since publication of the articles of Harrison and Kreps and of Harrison and Pliska. Indeed, truncated Lévy processes are not stable¹⁶ and not often infinitely divisible because the truncation of the distribution is abrupt¹⁷.

This problem of not-infinite-divisibility also implies a very large economic problem because it means that truncated Lévy processes are not continuous processes and therefore cannot be applied in situations of complete markets (Naik and Lee 1990). The theoretical link between completeness of market and continuous processes was developed by Harrison and Kreps (1979) and Harrison and Pliska (1981). In this perspective, the compound Poisson process cannot describe complete markets as defined by Arrow and Debreu (1954)¹⁸. This property has important consequences since it allows the possibility of having a unique price for contingent claims (like options). The incompleteness of markets implies a non-unique probability measure and therefore stochastic price processes take the form of a martingale, thus requiring a mathematical condition related to the existence of an arbitrage free market. Consequently, the development of

¹⁶ Only non-truncated Lévy processes are stable [59].

¹⁷ This implies that its shape is changing at different time horizons and that distribution at different time horizons do not obey scaling relations. Indeed, the variable x progressively converges towards a Lévy distribution for $x < N^*$ while it converges toward to a Normal distribution when x is beyond the crossover value N^* . More precisely, scaling turns out to be approximate and valid for a finite time interval only. For longer time intervals, scaling must break down. Moreover, some physicists have claimed that an abrupt truncation is only useful for very specific cases, but that this methodology would not be sufficiently physically plausible.

¹⁸ More precisely, Harrison and Kreps (1979) and Harrison and Pliska (1981) showed how a process must be continuous in order to obtain the uniqueness of the financial price.

exponentially truncated Lévy processes allows the emergence of a transeconophysics because, in opposition to truncated Lévy processes, they are continuous processes¹⁹ (implying therefore the uniqueness of prices)²⁰. Therefore, exponentially truncated Lévy processes could allow the creation of a common theoretical framework that transcends both fields.

Exponentially truncated Lévy distributions were introduced by Koponen (1995). He considered a truncated Lévy process in which the cut-off is a decreasing exponential function. Gupta and Campanha (1999) generalized this approach in econophysics by the following probability distribution,

$$P(x) = \begin{cases} L(x)_{(\alpha, \beta, \gamma, \delta)} & |x| > l \\ e^{-f(t,l)} L(x)_{(\alpha, \beta, \gamma, \delta)} & |x| \leq l \end{cases}$$

where l is the cut-off at which the distribution begins to deviate from Lévy distribution. $e^{-f(t,l)}$ is a decreasing function depending on time (t) and the cut-off parameter(s) (l) Like abruptly truncated Lévy distributions, exponentially truncated Lévy distributions have finite variance but their advantage is that they

¹⁹ They approach the continuous limit, being composed of an infinite number of small jumps in each time interval. This feature would make it possible to link these processes with research into the uniqueness of option prices.

²⁰ Lévy stable processes are a specific case of pure-jump processes and, as noted earlier, it is generally accepted that a pure-jump process corresponds to an incomplete market (Nolan 2009). However, a stable Lévy framework is not a strictly compound jump-process. Carr and Wu (2003, 754) explained that “in contrast to a standard Poisson or compound Poisson process, this pure jump process [stable Lévy process] has an infinite number of jumps over any time interval, allowing it to capture the extreme activity traditionally handled by diffusion processes. Most of the jumps are small and may be regarded as approximating the transition from one decimalized price to another one nearby”. In this perspective, stable Lévy processes can be seen as quasi-continuous processes as also emphasized by Nolan (2009).

are infinitely divisible, therefore making it possible to create a common theoretical framework based on uniqueness of the financial prices. In other words, exponentially truncated Lévy distribution could permit a generalization of the statistical framework used by financial economists.

Towards a financial economic interpretation of mathematical models coming from econophysics

The theoretical generalization of the statistical framework used by financial economists is an important step in a more integrative econophysics. However, a transeconophysics would need to develop an economic meaning for this generalization. From this perspective, Lévy processes and their parameters must have economic implications that will provide a common framework. Surprisingly, work on this issue does not even exist in econophysics. In the rest of this section we show that financial concepts of risk can be reinterpreted in such a way that they can fit into the framework suggested by stable Lévy processes.

The general form of Lévy distributions $S_{\alpha,\beta}(\gamma, \delta)$ can be characterized by four parameters. Depending on the value of these four parameters, two categories of Lévy processes can be identified: stable and non-stable. As we mentioned, econophysicists mainly use stable Lévy processes. A random variable X is said to be α -stable if we have specific values for parameters $(\alpha, \beta, \gamma, \delta)$ such as $0 < \alpha \leq 2$, $\gamma \geq 0$, $-1 \leq \beta \leq 1$, $\gamma \in \mathfrak{R}$. All the usual stable distributions (Gaussian,

Cauchy, Poisson, etc.) can then be found depending on the value of each parameter (α , β , γ , δ). Financial economics focuses on the mean (which characterizes return) and the variance (which characterizes risk). This discipline usually defines market risk as the dispersion of unexpected outcomes due to financial-market movements. From a statistical point of view, this dispersion is the result of the volatility and then the variance of the financial distributions. We consider that stable Lévy processes make it possible to complete the definition of traditional financial risk (volatility). Several papers are devoted to the analysis of these statistical parameters, but focus on variance or skewness only. In this perspective, we will propose to interpret each of the four parameters in terms of financial economics' theoretical framework, and more precisely, in terms of financial risk.

The *parameter* α is called the “characteristic exponent” and it shows the index of stability of the distribution. This value of this exponent determines the shape of the distribution: the lower this exponent, the fatter the tails (extreme events then have a higher probability of occurring). In other words, the lower α is, the more often extreme events are observed. In financial terms, then, this parameter is an indicator of risk since it describes how often important variations can occur. More precisely, α makes it possible to decompose the risk of a process into a “risk of shape.” This risk is often neglected²¹ because it is often associated with

²¹ Even when financial economists use models derived from ARCH (EGARCH, GARCH, etc.) to capture the leptokurticity of financial distributions, they implicitly assume that these distributions are Gaussian. Moreover, although, the standard ARCH model used in finance can reproduce the

traditional Gaussian distributions²².

The *parameter* γ is the scale indicator, which can be any positive number. It indicates the “random size,” i.e. the size of the variance whose regularity is given by the exponent α . This parameter describes the size of the variations (whose regularity is given by α). In a sense, γ can be seen as an indicator of statistical dispersion because it provides information about the potential maximum loss. It allows the risk of a process to be decomposed into a “size risk”. This risk is well known to financial economists, since this statistical dispersion is supposed to be reduced through diversification.

The two last statistical parameters, skeweness and localization factor, have been the focus of greater study in the financial literature.

The *parameter* β is an index of skewness: it gives information about the symmetry of the distribution and then about the place on the distribution where the most extreme value can be found. If $\beta = 0$, the distribution is symmetric. If $\beta < 0$ it is skewed toward the left (totally asymmetric toward the left if $\beta = -1$) meaning that most values are concentrated on the left of the distribution but that the most

power-law distribution of returns, they assume finite memory on past events and hence they are not consistent with long-range correlations in volatility observed on the market.

²² Financial economists usually use the concept of kurtosis to describe the leptokurticity of a distribution. However, this statistical concept is a Gaussian parameter (Balanda and MacGillivray 1988) and it often underestimates the leptokurticity in comparison with observed results (Tankov 2004).

extreme values are on the right. When $\beta > 0$ indicates a distribution skewed toward the right (totally asymmetric toward the left if $\beta = -1$), with most values concentrated on the right but the most extreme values on the left of the statistical distribution. This parameter is already well known in financial economics, since many works are devoted to the analysis of the skewness of financial distributions (see, for example, Rubinstein (1973), Sears and Wei (1985) or Harvey and Siddique (2000)).

Finally, the *parameter* δ a localization factor: it shifts the distribution right if $\delta > 0$ and left if $\delta < 0$. This factor describes the concentration of the data. It could be the mean of the median or other parameters that could describe the general shape of the distribution. This factor is not really a measure of risk since it describes what are the more frequently observed data. In this perspective, δ is rather a descriptive feature of the distribution.

Conclusion

This paper has dealt with the disciplinary dimension of econophysics. After showing that econophysics is a multidisciplinary field (rather than an interdisciplinary as several authors often claim), we posited the possibility of a transdisciplinary field between financial economics and econophysics.

We show that the emergence of a transeconophysics is possible through a common framework that would result, on the one hand, from the adaptation of physics models to theoretical constraints observed in financial economics and, on the other hand, from the interpretation of the financial concepts in terms of physics models. The first condition for a transdisciplinary econophysics is thus met. However, this step is far from being enough since these theoretical links between econophysics and financial economics do not guarantee real collaboration between the two communities²³.

²³ See Gingras and Schinckus (2012) for a bibliometric study of these two communities.

References

- (NSERC), Natural Sciences and Engineering Research Council of Canada. 2004. *Guidelines for the Preparation and Review of Applications in Interdisciplinary Research*. Ottawa: NSERC.
- Arrow, Kenneth Joseph and Gérard Debreu. 1954. Existence of an Equilibrium for a Competitive Economy. *Econometrica* 22. 3: 265-290.
- Aste, Tomaso and Tiziana Di Matteo. 2009. Introduction to Complex and Econophysics Systems: A navigation map. In *Complex physical, biophysical and econophysical systems*, edited by Dewar, Reward and Frank Detering, Canberra: World Scientific Book.
- Balanda, Kevin P. and H.L. MacGillivray. 1988. Kurtosis: A Critical Review. *The American Statistician* 42. 2: 111-119.
- Bernard-Bonnin, Anne-Claude, Sylvie Stachenko *et al.* 1995. Self-management teaching programs and morbidity of pediatric asthma: a meta-analysis. *Journal of Allergy and Clinical Immunology* 95. 1: 34-41.
- Bouchaud, Jean-Philippe 2002. An Introduction to statistical finance *Physica A* 313. 1: 238-251.
- Burda, Zdzislaw, Jerzy Jurkiewicz *et al.* 2003. Is Econophysics a Solid Sciences? *Acta Physica Polonica B* 34. 1: 87.
- Carbone, A., G. Kaniadakis *et al.* 2007. Where do we stand on econophysics. *Physica A* 382. 11-14.
- Carr, Peter and Liuren Wu. 2003. The Finite Moment Log Stable Process and Option Pricing. *The Journal of Finance* 58. 2: 753-777.
- Chatterjee, Arnab, Sudhakar Yarlagaadda *et al.* (eds.). 2005. *Econophysics of Wealth Distributions*. Milan: Springer.
- Choi, Bernard and Anita Pak. 2006. Multidisciplinarity, interdisciplinarity and transdisciplinarity in health research, services, education and policy: 1. Definitions, objectives, and evidence of effectiveness. *Clinical and Investigative Medicine* 29. 6: 351-364.
- D'Amour, D. and I. Oandasan. 2005. Interprofessionality as the field of interprofessional practice and interprofessional education: An emerging concept. *Journal of Interprofessional Care* 19. 1: 8-20.
- Daniel, Gilles and Didier Sornette. 2010. Econophysics: historical perspectives. In *Encyclopedia of Quantitative Finance*, edited by Rama, Cont, London: Wiley.
- Fama, Eugene F. 1963. Mandelbrot and the Stable Paretian Hypothesis. *Journal of Business* 36. 4: 420-429.
- . 1965. Portfolio Analysis in a Stable Paretian Market. *Management Science* 11. 3, Series A, Sciences: 404-419.
- Flinterman, Francesca, Rebecca Tecler-Mesbah *et al.* 2001. Transdisciplinarity: The new challenge for biomedical research. *Bulletin of Science, Technology & Society* 21. 4: 253-266.
- Gallegati, Mauro, Steve Keen *et al.* 2006. Worrying trends in econophysics. *Physica A* 370. 1: 1-6.
- Gingras, Yves and Christophe Schinckus. 2012. Institutionalization of Econophysics in the shadow of physics. *Journal of the History of Economic Thought* forthcoming.

- Gligor, Mircea and Margareta Ignat. 2001. Econophysics – a new field for statistical physics? *Interdisciplinary Science Reviews* 26. 4: 183-190.
- Grossman, Dave. 1979. *A quantitative system for the assessment of initial organizational needs in transdisciplinary research*. Michigan State University: East Lansing, Michigan.
- Gunaratne, Gemunu H. and Joseph L. McCauley. 2002. A theory for fluctuations in stock prices and valuation of their options. *Mathematical Modelling and Industrial Mathematics Papers* 2. 5: 4.
- . 2005. A theory for fluctuations in stock prices and valuation of their options. *Proceedings of the SPIE* 5848. 131.
- Gupta, Hari and José Campanha. 1999. The gradually truncated Lévy flight for systems with power-law distributions. *Physica A* 268. 231-239:
- Gupta, Hari and José Campanha. 2002. Tsallis statistics and gradually truncated Lévy flight - distribution of an economic index. *Physica A* 309. 4: 381-387.
- Harrison, J. Michael and David M. Kreps. 1979. Martingales and arbitrage in multiperiod securities markets. *Journal of Economic Theory* 20. 3: 381-408.
- Harrison, J. Michael and Stanley R. Pliska. 1981. Martingales and Stochastic Integrals in the Theory of Continuous Trading. *Stochastic Processes and their Applications* 11. 3: 215-260.
- Harvey, Campbell R. and Akhtar Siddique. 2000. Conditional skewness in asset pricing tests. *The Journal of Finance* 55. 3: 1263-1295.
- Ingrao, Bruna and Israel Giorgio. 1990. *The invisible hand: Economic equilibrium in the history of science* London: MIT Press.
- Israel, Giorgio. 2005. The science of complexity: epistemological problems and perspectives. *Science in Context* 18. 1-31.
- Jovanovic, Franck. 2006. A nineteenth-century random walk : Jules Regnault and the origins of scientific financial economics. In *Pioneers of Financial Economics, vol. I*, edited by Poitras, Geoffrey, 191-222. Cheltenham: Edward Elgar.
- (eds.). 2009. *L'institutionnalisation de l'économie financière : perspectives historiques*. Revue d'Histoire des Sciences Humaines.
- . 2010. Efficient markets theory. In *Encyclopedia of Quantitative Finance*, edited by Cont, Rama, Chichester (UK): John Wiley & Sons.
- Jovanovic, Franck and Philippe Le Gall. 2001. Does God practice a random walk? The "financial physics" of a 19th century forerunner, Jules Regnault. *European Journal for the History of Economic Thought* 8. 3: 323-362.
- Jovanovic, Franck and Christophe Schinckus. 2010a. *Econophysics : A New Challenge for Financial Economics ?*
- . 2010b. *Financial Economics' birth in 1960s*.
- . 2013. The History of Econophysics' Emergence: a New Approach in Modern Financial Theory. *History of Political Economy* 45. 2: forthcoming.
- Keen, Steve. 2003. Standing on the toes of pygmies: why econophysics must be careful of the economic foundations on which it builds. *Physica A* 324. 1: 108-116.
- Klein, Julie Thompson, and 1994. Notes Toward a Social Epistemology of Transdisciplinarity. Convento da Arrabida, Portugal:
- Klein, Julie Thompson 1990. *Interdisciplinarity: History, theory and practice*. Detroit: Wayne State University Press.

- Koponen, Ismo. 1995. Analytic approach to the problem of convergence of truncated Lévy flights towards the Gaussian stochastic process. *Physical Review E* 52. 1: 1197-1199.
- Kutner, Ryszard and Dariusz Grech. 2008. Report on Foundation and Organization of Econophysics Graduate Courses at Faculty of Physics of University of Warsaw and Department of Physics and Astronomy of the Wrocław University. *Acta Physica Polonica A* 114. 3: 637-647.
- Mandelbrot, Benoit. 1963. The Variation of Certain Speculative Prices. *Journal of Business* 36. 4: 394-419.
- . 1965. Very long-tailed probability distributions and the empirical distribution of city sizes. In *Mathematical Explanations in Behavioral Science*, edited by Massarik, F. and P. Ratoosh, 322-332. New York: Homewood Editions.
- . 1997. *Fractals, Hasard et Finance*. Paris: Flammarion.
- Mandelbrot, Benoit and Richard L. Hudson. 2004. *The (Mis)Behavior of Markets: A Fractal View of Risk, Ruin, and Reward*. London: Profile Books.
- Mantegna, Rosario N. and Eugene H. Stanley. 1994. Stochastic Process with Ultra-Slow Convergence to a Gaussian: The Truncated Lévy Flight. *Physical Review Letters* 73. 22: 2946-2949.
- . 1999. *An introduction to Econophysics*. New York: Cambridge University Press.
- Mariani, M. C. and Y. Liu. 2007. Normalized truncated Lévy walks applied to the study of financial indices. *Physica A* 377. 590-598.
- Markowitz, Harry M. 1952. Portfolio Selection. *Journal of Finance* 7. 1: 77-91.
- . 1959. *Portfolio selection; efficient diversification of investments*. New York: Wiley.
- Max-Neef, Manfred. 2005. Foundations of transdisciplinarity. *Ecological Economics* 53. 5-16.
- McCauley, Joseph L. 2004. *Dynamics of Markets : Econophysics and Finance*. Cambridge: Cambridge University Press.
- . 2006. Response to "Worrying Trends in Econophysics". *Physica A* 371. 1: 601-609.
- Mirowski, Philip. 1989. *More heat than light : economics as social physics, physics as nature's economics*. Cambridge ; New York: Cambridge University Press.
- Morin, Edgar, and 1994. Sur l'interdisciplinarité. Convento da Arrabida, Portugal:
- Naik, Vasanttilak and Moon Lee. 1990. General Equilibrium Pricing of Options on the Market Portfolio with Discontinuous Returns. *The Review of Financial Studies* 3. 4: 493-521.
- Nolan, John P. 2009. *Stable Distributions: Models for Heavy Tailed Data*. working paper of American University.
- Okuyama, K., M. Takayasu *et al.* 1999. Zipf's Law in Income distribution of Companies. *Physica A* 269. 1: 125-131.
- Rickles, Dickens. 2007. Econophysics for Philosophers *Studies in History and Philosophy of Science* 38. 4: 948-978.
- Roehner, Bertrand. 2002. *Patterns of speculation : a study in observational econophysics* Cambridge: Cambridge University Press.
- Rosenfield, P. L. 1992. The potential of transdisciplinary research for sustaining and

- extending linkages between the health and social sciences. *Social Science and Medicine* 35. 11: 1343-57.
- Rosser, Barkley. 2003. *Complexity in Economics*. New York: Edward Elgar.
- . 2006. The nature and future of econophysics. In *Econophysics of Stock and other Markets*, edited by Chatterjee, Arnab and Bikas K. Chakrabarti, 225-234. Milan: Springer.
- . 2008a. Econophysics and Economics Complexity *Advances in Complex Systems* 11. 5: 745-760.
- . 2008b. Debating the Role of Econophysics. *Nonlinear Dynamics, Psychology & Life Sciences* 12. 3: 311-323.
- Rubinstein, Mark. 1973. The Fundamental Theorem of Parameter.–preference Security Valuation. *Journal of Financial and Quantitative Analysis* 8. 1: 61-69.
- Samuelson, Paul A. 1965. Proof that properly anticipated prices fluctuate randomly. *Industrial Management Review* 6. 2: 41-49.
- Săvoiu, Gheorghe and Ion Iorga–Simăn. 2008. Some Relevant Econophysics' Moments of History, Definitions, Methods, Models and New Trends. *Romanian Economic and Business Review* 3. 3: 29-41.
- Schabas, Margaret. 1990. *A world ruled by number : William Stanley Jevons and the rise of mathematical economics*. Princeton, N.J.: Princeton University Press.
- Schinckus, Christophe. 2010. Econophysics and economics: sister disciplines? *American Journal of Physics* 78. 1: 325-327.
- Sears, R. Stephen and K.C. John Wei. 1985. Asset pricing, higher moments and the market risk premium: A note. *The Journal of Finance* 40. 4: 1251-1253.
- Stanley, Eugene H., Xavier Gabaix *et al.* 2008. A statistical physics view of financial fluctuations : Evidence for scaling universality. *Physica A* 387. 1: 3967-3981
- Stanley, Eugene H., Viktor Afanasyev *et al.* 1996. Anomalous fluctuations in the dynamics of complex systems: from DNA and physiology to econophysics. *Physica A* 224. 1: 302-321.
- Stember, Marilyn. 1998. Advancing the social sciences through interdisciplinary enterprise. In *Interdisciplinarity: Essays from the Literature*, edited by Newell, W., New York: College Board Publications.
- Tankov, Peter. 2004. *Processus de Lévy en Finance : Problèmes Inverses et Modélisation de dépendance*. Ecole Polytechnique de Paris: Paris.
- Vasconcelos, Giovanni. 2004. A Guided Walk Down Wall Street : An Introduction to Econophysics *Brazilian Journal of Physics* 34. 3: 1039-1065.
- Whitfield K, Reid C. 2004. Assumptions, ambiguities, and possibilities in interdisciplinary population health research. *The Canadian Journal of Public Health* 95. 6: 434-436.